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Function projective synchronization in coupled chaotic systems

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ABSTRACT

In this paper, the function projective synchronization is investigated in coupled partially linear chaotic systems. By Lyapunov stability theory, a control law is derived to make the state vectors asymptotically synchronized up to a desired scaling function. Furthermore, based on function projective synchronization, a scheme for secure communication is presented in theory. The corresponding numerical simulations are performed to verify and illustrate the analytical results.

1. Introduction

Since the pioneering work of Pecora and Carroll, which proposed a successful method to synchronize two identical chaotic systems with different initial conditions, chaos synchronization has become an active research subject in nonlinear science. Various types of chaos synchrony have been revealed to investigate chaos synchronization which include complete synchronization [1], phase synchronization [2], generalized synchronization [3], lag synchronization [4], and projective synchronization [5], etc. Amongst all kinds of chaos synchronization, projective synchronization has been extensively investigated [6–15] in recent years because it can obtain faster communication with its proportional feature. In application to secure communications, the proportional feature can be used to extend binary digital to M-nary digital communication [16] for getting faster communication.

Most of research efforts mentioned above have concentrated on studying the constant scaling factor. As compared with projective synchronization, function projective synchronization means that the master and slave systems could be synchronized up to a scaling function, but not a constant. This feature could be used to get more secure communications in application to secure communications [16–19], because the unpredictability of the scaling function in FPS can additionally enhance the security of communication. FPS is the more general definition of projective synchronization. FPS was first reported in Ref. [20], in which the authors only gave the FPS of the coupled Lü systems by the backstepping method. To the best of our knowledge, at present, there are few theoretical results about FPS. Motivated by the aforementioned reasons, this paper investigates a general scheme of FPS in coupled partially linear chaotic systems, and presents a secure communication scheme in theory.

The organization of this paper is as follows: In Section 2, the definition of FPS is given. In Section 3, a general scheme of FPS for coupled partially linear chaotic systems is presented. In Section 4, a numerical example is used to verify the effectiveness of the proposed scheme. In Section 5, a scheme for secure communication using FPS is presented in theory. The conclusion is finally drawn in Section 6.

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2. The definition of FPS

The drive system and the response system are defined as follows

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \tag{1}$$

$$\dot{\mathbf{y}} = \mathbf{g}(\mathbf{y}) + \mathbf{u}(\mathbf{x}, \mathbf{y}), \tag{2}$$

where $\mathbf{x}, \mathbf{y} \in \mathbf{R}^n$ are the state vectors, $\mathbf{f}, \mathbf{g} : \mathbf{R}^n \to \mathbf{R}^n$ are differentiable functions, $\mathbf{u}(\mathbf{x}, \mathbf{y})$ is a control function. We define the error term

$$\mathbf{e} = \mathbf{x} - \alpha(t)\mathbf{y}, \tag{3}$$

where $\alpha(t)$ is a continuously differentiable function with bounded, and $\alpha(t) \neq 0$ for all t.

Definition 1 (*FPS*). For the drive system (1) and the response system (2), it is said that the system (1) and the system (2) are function projective synchronization (FPS) if there exists a scaling function $\alpha(t)$, such that $\lim_{t\to\infty} \|\mathbf{e}(t)\| = \mathbf{0}$.

Remark 1. It is easy to see that the definition of function projection synchronization encompasses the projective synchronization when the scaling function $\alpha(t)$ is taken by a constant α.

3. A general method for FPS

A general form of partially linear system can be expressed by a set of differential equations

$$\begin{cases} \dot{\mathbf{u}} = \mathbf{M}(z)\mathbf{u}, \\ \dot{z} = g(\mathbf{u}, z), \end{cases} \tag{4}$$

in which the state vector can be broken into two parts $(\mathbf{u}, z)$. The state vector $\mathbf{u}$ has a linear form related to its time derivative $\dot{\mathbf{u}}$. The variable z is treated as the coupling variable that is nonlinearly related to the other variables in $\mathbf{u}$. The Jacobian matrix $\mathbf{M}(z) \in \mathbf{R}^{n\times n}$ is only dependent on the variable z.

The coupled systems through the variable z with a controller $\boldsymbol{\xi}$ can be expressed in the form as

$$\begin{cases} \dot{\mathbf{u}}_{\mathbf{d}} = \mathbf{M}(z)\mathbf{u}_{\mathbf{d}}, \\ \dot{z} = g(\mathbf{u}_{\mathbf{d}}, z), \\ \dot{\mathbf{u}}_{\mathbf{r}} = \mathbf{M}(z)\mathbf{u}_{\mathbf{r}} + \boldsymbol{\xi}. \end{cases} \tag{5}$$

The subscript of d denotes the drive system and r denotes the response system. If there exists a scaling function $\alpha(t)$ such that $\lim_{t\to\infty} \|\mathbf{u}_{\mathbf{d}} - \alpha(t)\mathbf{u}_{\mathbf{r}}\| = \mathbf{0}$, then the FPS between the drive system and response system is achieved.

Theorem 1. *For given synchronization scaling function $\alpha(t)$ and any initial conditions $\mathbf{u}_{\mathbf{d}}(0), z(0), \mathbf{u}_{\mathbf{r}}(0)$, the FPS between the two identical chaotic system (4) in the form as (5) will occur by the control law (6) as below*

$$\boldsymbol{\xi} = \frac{1}{\alpha(t)}\left[\mathbf{M}(z)\mathbf{e} - \dot{\alpha}(t)\mathbf{u}_{\mathbf{r}} + \mathbf{e}\right]. \tag{6}$$

Proof. We define the error vector as

$$\mathbf{e} = \mathbf{u}_{\mathbf{d}} - \alpha(t)\mathbf{u}_{\mathbf{r}}. \tag{7}$$

The time derivative of Eq. (7) is

$$\dot{\mathbf{e}} = \dot{\mathbf{u}}_{\mathbf{d}} - \alpha(t)\dot{\mathbf{u}}_{\mathbf{r}} - \dot{\alpha}(t)\mathbf{u}_{\mathbf{r}}. \tag{8}$$

Substituting (5) into (8), we have

$$\dot{\mathbf{e}} = \mathbf{M}(z)\mathbf{e} - \alpha(t)\boldsymbol{\xi} - \dot{\alpha}(t)\mathbf{u}_{\mathbf{r}}. \tag{9}$$

Construct dynamical Lyapunov function

$$V = \frac{1}{2}\mathbf{e}^{\mathrm{T}}\mathbf{e}. \tag{10}$$

The time derivative of V along the trajectories of Eq. (9) is

$$\begin{aligned} \dot{V} &= \mathbf{e}^{\mathrm{T}}\dot{\mathbf{e}} \\ &= \mathbf{e}^{\mathrm{T}}\left[\mathbf{M}(z)\mathbf{e} - \alpha(t)\boldsymbol{\xi} - \dot{\alpha}(t)\mathbf{u}_{\mathbf{r}}\right]. \end{aligned} \tag{11}$$