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Anti-synchronization of four scroll attractor with fully unknown parameters

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ABSTRACT

We have observed the anti-synchronization phenomena in coupled identical chaotic dynamical systems. Anti-synchronization can be characterized by the vanishing of the sum of relevant variables. Anti-synchronization problem of coupled identical chaotic dynamical systems with fully unknown parameters is analyzed. This technique is applied to achieve anti-synchronization of four-scroll atractor. Numerical simulations are provided to verify the effectiveness of the proposed method.

1. Introduction

Since the idea of synchronizing chaotic systems was introduced by Pecora and Carroll [1], there has been particular interest in chaotic synchronization, mainly due to its potential applications in secure communication, ecological systems, system identification, etc.

Even though the synchronization phenomenon of periodic oscillators has been known for a long time, those of chaotic systems are one of the most recent findings in nonlinear dynamics [2,3]. Various synchronization phenomena are being reported for coupled chaotic oscillators. Among them are complete synchronization (CS), phase synchronization (PS), lag synchronization (LS), and generalized synchronization (GS). CS is characterized by the convergence of two chaotic trajectories and has been observed in mutually coupled, uni-directionally coupled, and even noise induced chaotic oscillators [4–6]. Phase synchronization is characterized by that the phase difference between two chaotic oscillators are locked within 2π, while their amplitudes remain chaotic and uncorrelated [7,8]. Lag synchronization is described as the coincidence of two chaotic trajectories with a constant time lag [9,10]. Finally, generalized synchronization implies the establishment of functional relations between master and slave oscillators [11–13]. Other distinct characterizations of synchronization have also been reported [14].

On the other hand, anti-synchronization (AS) is another noticeable phenomenon in periodic oscillators that has been known for quite a long time. It is well known that the first observation of synchronization of two oscillators by Huygens in the seven-tenth century was, in fact, AS between two pendulum clocks. Recent re-investigation of Huygens' experiment by Blekhman [15] shows that either synchronization or AS can appear depending on the initial conditions of the coupled pendula. Here, AS can also be interpreted as anti-phase synchronization (APS) [16–18]. That is to say, there is no difference between AS and APS for oscillators with identical amplitudes [19]. AS phenomena have been observed experimentally in the context of self-synchronization, e. g., in salt-water oscillators [20–28], and some biological systems where a nonchaotic signal is generated. Moreover, it has been reported that APS can occur theoretically in a subsystem of hyperchaotic systems with symmetry [29].

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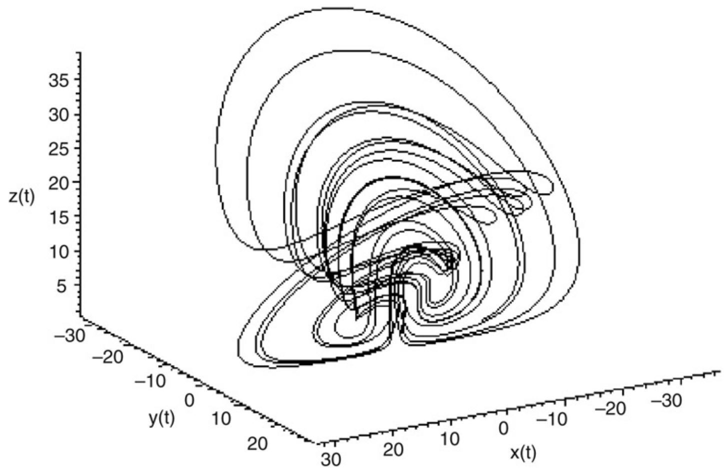

Fig. 1. Shows the chaotic attractor of tow-scroll attractor at $a = 4.5$, $b = 12$ and $c = 5$ in 3-dimensional.

In this paper, based on state observer [16–18], anti-synchronization is analyzed. The aim of this Letter is to further develop the state observer method for constructing a synchronized slave system. Finally, simulations on four scroll attractor are performed to verify the effectiveness and feasibility of the proposed control technique.

2. Systems description and mathematical models

We consider the drive chaotic system in the form of

$$\dot{x} = f(x, \alpha) \tag{1}$$

where $x \in R^n$ is the state vector of the system, $\alpha \in R^m$ is the parameter vector. $f \in C^1(R^n \times R^m)$ is nonlinear function. The response system is given by

$$\dot{y} = f(y, \alpha_1) \tag{2}$$

which has the same structure as the drive system.

Therefore, the goal of control is to design and implement an appropriate controller U for the response system and a parameter adaptive estimation law of the parameter, such that the controlled response system

$$\dot{y} = f(y, \alpha_1) + U \tag{3}$$

could be anti-synchronous with the drive system. This occurs iff

$$\operatorname*{Lim}_{t\to\infty}(y + x) = 0 \quad \text{and} \quad \operatorname*{Lim}_{t\to\infty}(\alpha_1 - \alpha) = 0. \tag{4}$$

Consider the following four-scroll attractor [30–32]:

$$\begin{aligned} \dot{x} &= ax - yz \\ \dot{y} &= -by + xz \\ \dot{z} &= -cz + xy \end{aligned} \tag{5}$$

where a, b and c are positive control parameters. This system exhibits a strange attractor at the parameter values $a = 0.4$, $b = 12$ and $c = 5$. This system bridges the gap between the Lorenz and Chen attractors [33–36].

Differing from other known similar systems, system (5) has five equilibria, and does not have Hopf and pitch bifurcations [31,32]. Of most interest is the observation that this chaotic system not only can display a tow-scroll chaotic attractor when $a = 4.5, b = 12$ and $c = 5$ (see Fig. 1), but also can display a four-scroll chaotic attractor when $a = 0.4, b = 12$ and $c = 5$ (see Fig. 2). Although, system (5) exhibits by computer simulation four-scroll chaotic attractor for certain values of the parameters no complete answer for the following challenging question. Is it true that a three-dimensional smooth quadratic autonomous system can generate a truly single four-scroll attractor?. On the other hand, from the engineering applications point of view, even a numerical four-scroll chaotic attractor can be quite useful due to its strong randomness and complex topological properties with a wider power spectrum. It implies that one can take advantage of this phenomena and use this kind of numerical chaotic signals for better and wider use in digital or electronic devices for some good engineering applications, such as random signal generation and secure communication.