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Bifurcation of a three molecular saturated reaction with impulsive input

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ABSTRACT

Although impulsive differential equations have become a widely concerned subject and a lot of models with impulsive effect have been studied in recent years, biochemical reaction models with impulsive input are rarely studied. In this paper, we consider an irreversible three molecular reaction model with impulsive input. By using the Floquet theorem and the method for the small parameter of impulsive differential equations, we obtain sufficient conditions for asymptotical stability and global stability of the given system. The existence of a positive periodic solution is also studied by the bifurcation theory. Further, we also show that our given conditions are right by numerical simulations.

1. Introduction

Most chemical reactions can present rich phenomena in vessel, such as chemical oscillations [1–5], period doubling, chemical waves [6,7], and chaos [8,9]. Analysis of forced nonlinear oscillations plays an important role in understanding their dynamic phenomena of electronic generators, mechanical, chemical and biological systems.

In the biochemical reaction, the reaction speed of the reactant can be proportional to its concentration, while there is no unlimited increase in the concentration of the reactant; when the concentration of the reactant reaches a certain value, the reaction speed of the reactant does not increase even if the concentration of the reactant increases. Let $V_{\max}$ denote the maximum specific growth rate, $[A]$ be the concentration of the reactant A, and K_m be the half saturation constant, then the relation between the reaction speed and the concentration of the reactant is

$$V = V_{\max}[A]/(K_m + [A]).$$

The irreversible three molecular saturated reaction process can be shown as follows:

$$X \to X + C, \qquad X + 2Y \to Y + D, \qquad c_1 \to Y, \qquad Y \xrightarrow{\frac{c_2 y}{k_1+y}} E \quad (\text{output}),$$

where X, Y are the concentrations of reaction products in time t, and C, D, E are reaction products. According to the above reaction process, we can write the following differential equations to study the irreversible three molecular saturated reaction:

$$\begin{cases} x'(t) = x(t)(1 - y^2(t)), \\ y'(t) = c_1 + x(t)y^2(t) - \dfrac{c_2 y(t)}{k_1 + y(t)}, \end{cases} \tag{1.1}$$

where $x(t), y(t)$ are the concentrations of reaction products in time t, and c_1, k_1, c_2 are positive constants.

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In this paper, we will consider an irreversible three molecular reaction model with impulsive input. This paper is organized as follows. The simply qualitative analysis of the system without impulsive effect is given in Section 2. In Section 3, asymptotical stability and global stability of the given system will be investigated. The existence of a positive periodic solution will be studied by the bifurcation theory in Section 4. Numerical simulations are given in Section 5. Finally, some conclusions and biological discussions are provided in Section 6.

2. Simple analysis of system (1.1)

Let the right of system (1.1) equal to zeros, we obtain its equilibriums $A\left(\frac{c_2-c_1(k_1+1)}{k_1+1}, 1\right)$, $B\left(0, \frac{c_1k_1}{c_2-c_1}\right)$ and $C\left(-\frac{c_2+c_1(k_1-1)}{k_1-1}, -1\right)$. If $0 < c_2 - c_1 < c_1k_1$, system (1.1) does not have any positive equilibrium point, and equilibrium point B is stable node. Next, if $c_2 > c_1(k_1 + 1)$, we easily know equilibrium point B is saddle and point A is focus (or node).

We now analyze equilibrium point A. Let

$$\begin{cases} x'(t) = x(t)(1 - y^2(t)) = P(x(t), y(t)), \\ y'(t) = c_1 + x(t)y^2(t) - \dfrac{c_2y(t)}{k_1 + y(t)} = Q(x(t), y(t)), \end{cases} \tag{2.1}$$

we have

$$q|_A = \frac{\partial(P, Q)}{\partial(x(t), y(t))}|_A = \begin{vmatrix} 0 & -2\left(\dfrac{c_2 - c_1 - c_1k_1}{k_1 + 1}\right) \\ 1 & \dfrac{c_2(k_1 + 2) - 2c_1(k_1 + 1)^2}{(k_1 + 1)^2} \end{vmatrix}$$
$$= 2\left(\frac{c_2 - c_1 - c_1k_1}{k_1 + 1}\right) > 0,$$

and

$$p|_A = -\frac{c_2(k_1 + 2) - 2c_1(k_1 + 1)^2}{(k_1 + 1)^2}.$$

We know that point A is asymptotically stable if $c_2(k_1 + 2) > 2c_1(k_1 + 1)^2$.

If $c_2(k_1 + 2) > 2c_1(k_1 + 1)^2$ holds, we have the following theorem:

Theorem 2.1 ([10]). *System (1.1) has the limit cycles surrounding the positive singular point A if and only if* $c_2(k_1 + 2) > 2c_1(k_1 + 1)^2$.

By Theorem 2.1, we know that system (1.1) has not a limit cycle if $c_2(k_1 + 2) \le 2c_1(k_1 + 1)^2$.

3. The model with impulsive input

Impulsive differential equations have become a widely concerned subject in recent years, for example [11–26]. In real life, the concentration of the reactant cannot be added all the time. Therefore, the impulse to add the concentration of reactant is in line with the actual situation. With an impulse perturbation, the irreversible three molecular saturated reaction model becomes:

$$\begin{cases} \left.\begin{aligned} x'(t) &= x(t)(1 - y^2(t)), \\ y'(t) &= x(t)y^2(t) - \frac{c_2y(t)}{k_1 + y(t)}. \end{aligned}\right\} t \neq nT, \\ \left.\begin{aligned} \Delta x(t) &= 0, \\ \Delta y(t) &= c_1. \end{aligned}\right\} t = nT \end{cases} \tag{3.1}$$

where $\Delta x(t) = x(t^+) - x(t)$, $\Delta y(t) = y(t^+) - y(t)$, c_1, c_2, k_1 are positive constant, $T > 0$ is pulse periodic. In order to facilitate the calculation, we let $\mathrm{d}t = (k_1 + y(t))\mathrm{d}\tau$ and still take τ as t, then system (3.1) is rewritten as:

$$\begin{cases} \left.\begin{aligned} x'(t) &= (k_1 + y(t))x(t)(1 - y^2(t)), \\ y'(t) &= k_1x(t)y^2(t) + x(t)y^3(t) - c_2y(t). \end{aligned}\right\} t \neq nT, \\ \left.\begin{aligned} \Delta x(t) &= 0, \\ \Delta y(t) &= c_1. \end{aligned}\right\} t = nT. \end{cases} \tag{3.2}$$