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Global exponential stability in Lagrange sense for recurrent neural networks with both time-varying delays and general activation functions via LMI approach

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ABSTRACT

In this paper, we study the global exponential stability in a Lagrange sense for recurrent neural networks with both time-varying delays and general activation functions. Based on assuming that the activation functions are neither bounded nor monotonous or differentiable, several algebraic criterions in linear matrix inequality form for the global exponential stability in a Lagrange sense of the neural networks are obtained by virtue of Lyapunov functions and Halanay delay differential inequality. Meanwhile, the estimations of the globally exponentially attractive sets are given out. The results derived here are more general than that of the existing reference. Finally, two examples are given and analyzed to demonstrate our results.

1. Introduction

In the past two decades, there has been increasing widespread concern on neural networks, due to their successful applications in many areas, such as pattern recognition, paralleled computations, associative memory and so on. As we know, the integration and communication delays are unavoidably encountered both in biological and artificial neural systems, which will may lead to poor performance such as oscillation, instability, chaos, etc. Hence, there are a large number of results on the stability in a Lyapunov sense for neural networks with bounded or unbounded time delays [1–10]. For example, based on the Lyapunov functional and the free-weighting matrix method, some sufficient conditions for the exponential stability in a Lyapunov sense of equilibrium of the neural network with time-varying delays and general activation functions are derived [1]. Cao et al. [2] studied the exponential stability in a Lyapunov sense for a class of high-order bidirectional associative memory neural networks with time delays, and presented several sufficient conditions which can ensure the system to be globally exponentially stable by employing the linear matrix inequality and the Lyapunov functional methods. It is worth mentioning that Lyapunov stability refers to the stability of equilibrium points which requires the existence of equilibrium points, while Lagrange stability refers to the stability of the total system which doesn't require the information of equilibrium points. Moreover, the global stability in a Lyapunov sense can be viewed as a special case of stability in a Lagrange sense by regarding an equilibrium point as an attractive set [11]. So it is necessary and rewarding to study Lagrange stability. Basically, the goal of the study on global stability in a Lagrange sense is to determine global attractive sets. Once a global attractive set is found, a rough bound of periodic states and chaotic attractors can be estimated. Therefore, a considerable number of works studied the Lagrange stability for neural networks with time-delays [11–19]. For instance, by constructing several proper Lyapunov functionals combining with Jensen's inequality, Itô 's formula and some analytic techniques, Wang et al. gave several sufficient conditions in linear matrix inequality forms for the global dissipativity in the mean of stochastic

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neural networks [12]. Liao et al. [13] studied the global exponential stability in a Lagrange sense for continuous recurrent neural networks (RNNs) with multiple time delays by constructing appropriate Lyapunov-like functions, and analyzed three different types of activation functions which include both bounded and monotonous nondecreasing active functions. To our best knowledge, few authors have discussed the stability in a Lagrange sense of neural networks with general activation functions which are neither bounded nor monotonous, and there are few results made on it by LMIs [12].

Motivated by the above analysis, the aim of this paper is to study the global exponential stability in a Lagrange sense and the existence of globally exponentially attractive(GEA) sets for recurrent neural networks with both time-varying delays and general activation functions. The remainder of this paper is organized as follows: Section 2 describes some preliminaries including some necessary definitions, assumptions and lemmas. The main results are stated in Section 3. Section 4 gives some numerical examples to verify our main results. Finally, conclusions are presented in Section 5.

2. Preliminaries

In this paper, $R^+ = (0, +\infty)$; R^n denotes the n-dimensional Euclidean space; $\mathbb{C}[X, Y]$ is a class of continuous mapping set from the topological space X to topological space Y. Especially, $\mathbb{C} \triangleq \mathbb{C}((t_0 - \tau, t_0], R^n)$.

Considering the following recurrent neural networks (RNNs)

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -Cx(t) + Af(x(t)) + Bf(x(t - \tau(t))) + U, \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$, $x_i(t)$ corresponds to the ith neuron at time t, $i \in \Gamma = \{1, 2, \ldots, n\}$; $C = \mathrm{diag}\{c_1, c_2, \ldots, c_n\}$, $c_i \in R^+$ is the self-feedback connection weight, $i \in \Gamma$; $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$ are connection weights related to the neurons without and with delays, respectively; $U = (u_1, u_2, \ldots, u_n)^T$ is an external input; $\tau(t) = (\tau_1(t), \tau_2(t), \ldots, \tau_n(t))^T$ which is the time-varying delay satisfies $0 < \tau_i(t) \le \tau_i$ (τ_i is a constant), $i \in \Gamma$, and $\tau = \max_{1\le i\le n}\{\tau_i\}$; $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T$, $f(x(t - \tau(t))) = (f_1(x_1(t - \tau_1(t))), f_2(t - \tau_2(t))), \ldots, f_n(t - \tau_n(t))^T$, $f_i(\cdot)$ are activation functions. And an equilibrium point of the system (1) is a constant vector $x^* = (x_1^*, x_2^*, \ldots, x_n^*)^T$ satisfying $-Cx^* + Af(x^*) + Bf(x^*) + U = 0$. In this paper, the matrix $A < 0$ denotes that the matrix A is negative definite and $A < B$ indicates $A - B < 0$. We make the following assumption:

(H) There exist two diagonal matrices $L = \mathrm{diag}\{L_1, L_2, \ldots, L_n\}$ and $F = \mathrm{diag}\{F_1, F_2, \ldots, F_n\}$ such that for any $x, y \in R$ and $x \ne y$, the following inequalities hold

$$L_i \le \frac{f_i(x) - f_i(y)}{x - y} \le F_i, \quad i \in \Gamma.$$

For any initial function $\varphi(s) \in \mathbb{C}, s \in [t_0 - \tau, t_0]$, the solution of (1) that starts from the initial condition φ will be denoted by $x(t, t_0, \varphi)$ or simply $x(t)$ if no confusion should occur.

In the remaining part of this section, we will give some definitions and lemmas so that our main conclusions can be expediently explained in the ensuing sections.

Definition 2.1 (*[13]*). The network (1) is said to be uniformly stable in a Lagrange sense (or uniformly bounded), if for any $\alpha > 0$, there exists a constant $K = K(\alpha) > 0$ such that $\|x(t; \varphi)\| < K$ for $\forall\varphi \in C_\alpha \triangleq \{\varphi \in \mathbb{C} \mid \|\varphi\| \le \alpha\}$ and $t \ge t_0$.

Definition 2.2 (*[11]*). A set $\Omega \subseteq R^n$ is said to be a attractive set of (1), if for $\forall s \in [t_0 - \tau(t_0), t_0]$, $x(s) \in R^n \setminus \Omega$, $\lim_{t\to+\infty}\rho(x(t), \Omega) = 0$ holds, where $R^n \setminus \Omega$ is the complement set of Ω, and $\rho(x, \Omega) = inf_{y\in\Omega}\|x - y\|$ is the distance between x and Ω.

Definition 2.3 (*[13]*). A compact set $\Omega \subseteq R^n$ is said to be a GEA set of (1) (in strong sense), if there exists a positive constant α, a nonnegative continuous function $K(\cdot)$ such that for any solution $x(t)$ with $x(t) \in R^n \setminus \Omega$, $t \ge 0$, we have

$$\rho(x(t), \Omega) \le K(\varphi) \exp(-\alpha t), \quad t \ge t_0.$$

Definition 2.4 (*[13]*). If there exists a radially unbounded and positive definite function $V(x)$, a nonnegative continuous function $K(\cdot)$, and two positive constants ℓ and α such that for any solution $x(t) = x(t, \varphi)$ of (1), $V(x(t)) > \ell$, implies

$$V(x(t)) - \ell \le K(\varphi) \exp\{-\alpha t\}, \quad t \ge t_0,$$

then the network (1) is said to be globally exponentially attractive with respect to V. The compact set $\Omega \triangleq \{x \in R^n | V(x) \le \ell\}$ is said to be a GEA set of (1).

Definition 2.5 (*[13]*). The network (1) is called globally exponentially stable (GES) in a Lagrange sense, if it is both uniformly stable in a Lagrange sense and globally exponentially attractive. If there is a need to emphasize the Lyapunov-like functions, the network will be called globally exponentially stable in a Lagrange sense with respect to V.

Obviously, if the network (1) has a global attractive set, it is ultimately bounded; and if the network (1) has a GEA set, it is GES in a Lagrange sense.

Lemma 2.1 ([5]). *Let $a, b \in R^n$, P be a positive definite matrix, then $2a^T b \le a^T P^{-1} a + b^T P b$.*