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Analysis of a stage-structured predator–prey system with birth pulse and impulsive harvesting at different moments[☆]

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ABSTRACT

In this work, we consider a stage-structured predator–prey system with birth pulse and impulsive harvesting at different moments. Firstly, we prove that all solutions of the investigated system are uniformly ultimately bounded. Secondly, the conditions of the globally asymptotically stable prey-extinction boundary periodic solution of the investigated system are obtained. Thirdly, the permanence of the investigated system is also obtained. Finally, numerical analysis is inserted to illustrate the results. Our results provide reliable tactic basis for the practical biological economics management.

1. Introduction

The predator–prey, competitive and cooperative models have been studied by many authors (see monographs [1–11] and papers [12,13]). The permanence and extinction are significant concepts of those models which also show many interesting results. However, the stage structure of species has been considered very little. In the real world, almost all animals have the stage structure of immature and mature. Recently, papers [14–16] studied the stage structure of species with or without time delays.

Therefore, we consider the stage structure of immature and mature of the population, their densities of the population are written as $x_1(t)$ and $x_2(t)$, respectively. It satisfies the following assumptions:

(H1): The birth rate of the immature population is proportional to the existing mature population with a proportionality constant b (that is the term $bx_2(t)$ in the first equation of System (1.1)); for the immature population, the death rate and transformation rate of the mature population are proportional to the existing immature population with proportionality constants c and d_1 (that is the terms $cx_1(t)$ and $d_1x_1(t)$ in the first equation of System (1.1)); the immature population is density restriction (that is the term $\eta x_1^2(t)$ in the first equation of System (1.1)).

(H2): The death rate of the mature population is proportional to the existing mature population with a proportionality constant d_2 (that is the term $d_2x_2(t)$ in the second equation of System (1.1)).

According to Assumptions (H1) and (H2), we set up the following stage-structured single population model

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$$\begin{cases} \dfrac{dx_1(t)}{dt} = bx_2(t) - (c+d_1)x_1(t) - \eta x_1^2(t), \\ \dfrac{dx_2(t)}{dt} = cx_1(t) - d_2x_2(t), \end{cases} \tag{1.1}$$

where b, c, d_1, d_2, η are positive constants.

Theories of impulsive differential equations have been introduced into population dynamics lately [17–19]. Impulsive equations are found in almost every domain of applied science and have been studied in many investigations [16,20,18, 21], they generally describe phenomena which are subject to steep or instantaneous changes. Jiao et al. [21] suggested that releasing pesticides is combined with transmitting infective pests into an SI model. This may be accomplished by avoiding periods when the infective pests would be exposed or placing the pesticides in a location where the transmitting infective pests would not contact it. So an impulsive differential equation modeling the process of releasing infective pests and spraying pesticides at different fixed moment was represented as

$$\begin{cases} \left.\begin{aligned} \frac{dS(t)}{dt} &= rS(t)\left(1 - \frac{S(t)+\theta I(t)}{K}\right) - \beta S(t)I(t), \\ \frac{dI(t)}{dt} &= \beta S(t)I(t) - I(t), \end{aligned}\right\} t \neq (n-1+l)\tau, t \neq n\tau, \\ \left.\begin{aligned} \triangle S(t) &= -\mu_1 S(t), \\ \triangle I(t) &= -\mu_2 I(t), \end{aligned}\right\} t = (n-1+l)\tau, n = 1, 2 \ldots, \\ \left.\begin{aligned} \triangle S(t) &= 0, \\ \triangle I(t) &= \mu, \end{aligned}\right\} t = n\tau, n = 1, 2 \ldots. \end{cases} \tag{1.2}$$

For the biological meaning of the parameters in System (1.2) we can refer to [21].

Clack [22] has studied the optimal harvesting of the logistic equation, a logistic equation without exploitation as follows

$$\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right), \tag{1.3}$$

where $x(t)$ represents the density of the resource population at time t, r is the intrinsic growth rate, the positive constant K is usually referred to as the environmental carrying capacity or saturation level. If the population described by logistic equation (1.3) is subjected to harvesting under the catch-per-unit effort hypothesis $h(t) = Ex(t)$, the equations of the harvested population can be revised as follows

$$\frac{dx(t)}{dt} = rx(t)\left(1 - \frac{x(t)}{K}\right) - Ex(t), \tag{1.4}$$

where E denotes the harvesting effort. Leung [23] has studied optimal harvesting-coefficient control of the predator–prey diffusive Volterra–Lotka system. Alvarez and Shepp [24] have studied optimal harvesting of stochastically fluctuating populations. Tang and Chen [25] have studied the effect of seasonal harvesting on stage-structured fishery models, and obtained some results about optimal harvesting policy for the models.

The organization of this paper is as follows. In the next section, we introduce the model and background concepts. In Section 3, some important lemmas are presented. In Section 4, we give the globally asymptotically stable conditions of prey-extinction periodic solution of System (2.1), and the permanent condition of System (2.1). In Section 5, numerical analysis and a brief discussion are given to conclude this work.

2. The model

In this work, we consider a stage-structured predator–prey model with birth pulse and impulsive harvesting on predator population at different moments

$$\begin{cases} \left.\begin{aligned} \frac{dx(t)}{dt} &= x(t)(a - bx(t)) - \beta x(t)y_1(t), \\ \frac{dy_1(t)}{dt} &= k\beta x(t)y_1(t) - (c+d_1)y_1(t), \\ \frac{dy_2(t)}{dt} &= cy_1(t) - d_2y_2(t), \end{aligned}\right\} t \neq (n+l)\tau, t \neq (n+1)\tau, \\ \left.\begin{aligned} \triangle x(t) &= -\mu_1 x(t), \\ \triangle y_1(t) &= 0, \\ \triangle y_2(t) &= -\mu_2 y_2(t), \end{aligned}\right\} t = (n+l)\tau, n = 1, 2 \ldots, \\ \left.\begin{aligned} \triangle x(t) &= 0, \\ \triangle y_1(t) &= y_2(t)(a_1 - b_1y_2(t)), \\ \triangle y_2(t) &= 0, \end{aligned}\right\} t = (n+1)\tau, n = 1, 2 \ldots, \end{cases} \tag{2.1}$$