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Almost periodic solutions of a discrete Lotka–Volterra competition system with delays

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ABSTRACT

In this paper, we consider a discrete almost periodic Lotka–Volterra competition system with delays. Sufficient conditions are obtained for the permanence and global attractivity of the system. Further, by means of an almost periodic functional hull theory, we show that the almost periodic system has a unique strictly positive almost periodic solution, which is globally attractive. Some examples are presented to verify our main results.

1. Introduction

In this paper, we study an almost periodic sequence solution of the following discrete Lotka–Volterra system with delays:

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left(r_1(n)\left(1-\frac{x_1(n-\tau_{11})}{K_1(n)}-\mu_2(n)x_2(n-\tau_{12})\right)\right),\\ x_2(n+1) &= x_2(n)\exp\left(r_2(n)\left(1-\mu_1(n)x_1(n-\tau_{21})-\frac{x_2(n-\tau_{22})}{K_2(n)}\right)\right), \end{aligned} \tag{1.1}$$

where $\{r_i(n)\}$, $\{K_i(n)\}$ and $\{\mu_i(n)\}$, $i = 1, 2$, are bounded nonnegative almost periodic sequences (see Section 2 for the definitions) such that

$$0 < r_i^L \le r_i(n) \le r_i^M, \qquad 0 < K_i^L \le K_i(n) \le K_i^M, \qquad 0 < \mu_i^L \le \mu_i(n) \le \mu_i^M,$$

where $i = 1, 2$ and $n \in Z$. Here, we denote by Z and Z^+ the sets of integers and nonnegative integers, respectively. For any bounded sequence $\{g(n)\}$ defined on Z, define $g^M = \sup_{n\in Z} g(n)$, $g^L = \inf_{n\in Z} g(n)$.

We consider system (1.1) with the following initial conditions:

$$\begin{aligned} &x(\theta) = \varphi(\theta) \ge 0, \qquad y(\theta) = \psi(\theta) \ge 0, \quad \varphi(0) > 0, \psi(0) > 0,\\ &\theta \in N[-\tau, 0] = \{-\tau, -\tau+1, \ldots, 0\}, \quad \tau = \max\{\tau_{ij}, i, j = 1, 2\}. \end{aligned}$$

It is not difficult to see that solutions of (1.1) are well defined for all $n \ge 0$ and satisfy $x_i(n) > 0$.

Recently, the existence and uniqueness of almost periodic solutions have been intensively investigated in many papers; see [1–3] and the references cited therein. In fact, it has been found that the dynamic behaviors of the discrete systems are

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rather complex and richer than those of the continuous ones [4]. Therefore, it is more realistic to consider dynamic systems governed by difference equations. For more details in this direction, please see [5–19].

Zhou and Zou [10] studied the following discrete periodic logistic equation:

$$x(n+1) = x(n)\exp\left(r(n)\left(1-\frac{x(n)}{K(n)}\right)\right). \tag{1.2}$$

Sufficient conditions for the persistence and existence of a stable periodic solution of system (1.2) were obtained. Further, under the assumptions of almost periodicity of the coefficients of system (1.2), Li and Chen [11] obtained the existence of a unique almost periodic solution which is globally attractive.

Chen and Zhou [12] discussed a discrete Lotka–Volterra competition system:

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left(r_1(n)\left(1-\frac{x_1(n)}{K_1(n)}-\mu_2(n)x_2(n)\right)\right),\\ x_2(n+1) &= x_2(n)\exp\left(r_2(n)\left(1-\mu_1(n)x_1(n)-\frac{x_2(n)}{K_2(n)}\right)\right). \end{aligned} \tag{1.3}$$

They obtained sufficient conditions which guarantee the persistence of system (1.3). Also, for the periodic case, they also obtained sufficient conditions which guarantee the existence of a globally stable periodic solution.

Niu and Chen [13] discussed a discrete almost periodic solution of a Lotka–Volterra competition system with feedback controls. They obtained the existence and uniqueness of the almost periodic solution which is uniformly asymptotically stable. Zhang et al. [14] discussed a discrete almost periodic fishing model with feedback control of the following form:

$$\begin{aligned} x(n+1) &= x(n)\exp\left(\frac{a(n)}{1+\left[\frac{x(n)}{K(n)}\right]^r}-b(n)-c(n)u(n)\right),\\ \Delta u(n) &= -\alpha(n)u(n)+\beta(n)x(n). \end{aligned} \tag{1.4}$$

Sufficient conditions were established for the persistence of the above system, and the existence and uniformly asymptotical stability of an almost periodic solution of this system were investigated. Li and Zhang [15] considered a discrete delay logistic equation with feedback control. Sufficient conditions which guarantee the permanence and existence of a unique globally attractive positive almost periodic sequence solution of the system were obtained.

In [17], a discrete population model with time delays was studied. Sufficient conditions are obtained to ensure a positive solution of the model that is stable and attracts all positive solutions. Muroya [18–20] considered the persistence and global stability of some discrete models of Lotka–Volterra type with delays. But to the best of the authors' knowledge, to this day, still no scholars have considered discrete almost periodic Lotka–Volterra competition systems with delays. Therefore, with stimulation from the works of [11,13–15,17–20], the main purpose of this paper is to derive a set of sufficient conditions ensuring the permanence and existence of a unique almost periodic solution of system (1.1).

The organization of this paper is as follows. In Section 2, we give some definitions and present some useful lemmas. Sufficient conditions for the permanence and global attractivity of system (1.1) are obtained in Section 3. Then, in Section 4, we establish sufficient conditions for the existence of a unique almost periodic solution, which is globally attractive. The main result is illustrated by giving some examples with numerical simulations in the last section.

2. Preliminaries

First, we give the definitions of the terminologies involved.

Definition 2.1 ([21]). A sequence $x : Z \to R$ is called an almost periodic sequence if the ε-translation set of x

$$E\{\varepsilon, x\} = \{\tau \in Z : |x(n+\tau) - x(n)| < \varepsilon, \forall n \in Z\}$$

is a relatively dense set in Z for all $\varepsilon > 0$; that is, for any given $\varepsilon > 0$, there exists an integer $l(\varepsilon) > 0$ such that each interval of length $l(\varepsilon)$ contains an integer $\tau \in E\{\varepsilon, x\}$ with

$$|x(n+\tau) - x(n)| < \varepsilon, \quad \forall n \in Z.$$

τ is called an ε-translation number of $x(n)$.

Definition 2.2 ([22]). Let D be an open subset of R^m, $f : Z \times D \to R^m$. $f(n, x)$ is said to be almost periodic in n uniformly for $x \in D$ if for any $\varepsilon > 0$ and any compact set $S \subset D$, there exists a positive integer $l = l(\varepsilon, S)$ such that any interval of length l contains an integer τ for which

$$|f(n+\tau, x) - f(n, x)| < \varepsilon, \quad \forall (n, x) \in Z \times S.$$

τ is called an ε-translation number of $f(n, x)$.