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Stability and bifurcation analysis of a discrete predator–prey model with nonmonotonic functional response☆

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ABSTRACT

The paper studies the dynamical behaviors of a discrete predator–prey system with nonmonotonic functional response. The local stability of equilibria of the model is obtained. The model undergoes flip bifurcation and Hopf bifurcation by using the center manifold theorem and the bifurcation theory. Numerical simulations not only illustrate our results, but also exhibit the complex dynamical behaviors of the model, such as the period-doubling bifurcation in periods 2, 4 and 8, and quasi-periodic orbits and chaotic sets. The most interesting aspect is choosing the same parameters and the initial value of the model; then we vary the parameter K, and obtain series bifurcations, such as flip bifurcation and Hopf bifurcation.

1. Introduction

As is well known, in the theory of population dynamical models there are two kinds of mathematical models: the continuous-time models described by differential equations, and the discrete-time models described by difference equations. In recent years, more and more attention is being paid to discrete-time population models. The reasons are as follows. First, the discrete-time models are more appropriate than the continuous-time models when populations have non-overlapping generations or the number of populations is small. Second, we can get more accurate numerical simulation results from discrete-time models. Moreover, the numerical simulations of continuous-time models are obtained by discretizing the models. At last, the discrete-time models have rich dynamical behaviors; for example, the single-species discrete-time models have bifurcations, chaos and more complex dynamical behaviors (see, [1]).

Predator–prey models have already received much attention from many authors. For example, the stability, permanence and the existence of periodic solutions of the predator–prey models are studied in [2–12]. For the continuous-time predator–prey models, many authors have chosen delay as the bifurcation parameter to discuss the Hopf bifurcation in [13–17]. Xiao, Li and Han discussed the B–T bifurcation and Hopf bifurcation of a ratio-dependent predator–prey model with predator harvesting in [18]. However, there are few articles discussing the dynamical behaviors of predator–prey models, which include bifurcations and chaos phenomena for the discrete-time models. Liu and Xiao [19] obtained the flip bifurcation and Hopf bifurcation by using the center manifold theorem and the bifurcation theory. But Agiza et al. [20] and Celik et al. [21] only showed the flip bifurcation and Hopf bifurcation by using numerical simulations.

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We consider the following continuous-time predator–prey model described by differential equations.

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = rx\left(1-\dfrac{x}{K}\right) - \dfrac{xy}{a+x^2}, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = y\left(\dfrac{\mu x}{a+x^2} - D\right), \end{cases} \tag{1.1}$$

where $x(t)$, and $y(t)$, denote the numbers of prey and predator at time t, respectively. $K > 0$ is the carrying capacity of the prey, $r > 0$ is the intrinsic growth rate, $\mu > 0$ is the conversion coefficient and $D > 0$ is the death rate of the predator, $a > 0$ is the half-saturation constant. Ruan and Xiao in [22] discussed the global qualitative analysis of model (1.1) depending on all parameters and showed that model (1.1) exhibited the Bogdanov–Takens bifurcation. By choosing the carrying capacity of the prey and the death rate of the predator as bifurcation parameters, it showed that model (1.1) undergoes a series of bifurcations including the saddle–node bifurcation, the supercritical and subcritical Hopf bifurcations, and the homoclinic bifurcation.

In this paper, motivated by the above works we study the following discrete-time model corresponding with model (1.1).

$$\begin{cases} x(n+1) = x(n)\exp\left[r\left(1-\dfrac{x(n)}{K}\right) - \dfrac{y(n)}{a+x(n)^2}\right], \\ y(n+1) = y(n)\exp\left[\dfrac{\mu x(n)}{a+x(n)^2} - D\right], \end{cases} \tag{1.2}$$

where r, a, μ, D, and K are defined as in model (1.1). It is assumed that the initial value of solutions in system (1.1) satisfied $x(0) > 0$, $y(0) > 0$ and all the parameters are positive. It is easy to prove that if the initial value $(x(0), y(0))$ is positive, then the corresponding solution $(x(n), y(n))$ is positive too.

In this paper, we will study the dynamical behaviors of model (1.2). By using the theory of difference equation, the theory of bifurcation and the center manifold theorem we will establish the series of criteria on the existence and local stability of equilibria, flip bifurcation and Hopf bifurcation. Furthermore, by using the numerical simulations method we will indicate the correctness and rationality of our results.

The organization of this paper is as follows. In the second section we discuss the existence and local stability of equilibria in model (1.2). In the third section we study flip bifurcation and Hopf bifurcation for model (1.2) by choosing K as a bifurcation parameter. In the fourth section we present the numerical simulations, which not only illustrate our results with the theoretical analysis, but also exhibit the complex dynamical behaviors such as the cascade of period-doubling bifurcation in periods 2, 4 and 8, and quasi-periodic orbits and chaotic sets . In the last section we give the discussion.

2. Analysis of equilibria

We firstly discuss the existence of the equilibria of model (1.2). Obviously, $E_0(0, 0)$ and $E_1(K, 0)$ are two equilibria of model (1.2). Furthermore, from model (1.2) we know the others equilibria of model (1.2) satisfy

$$\begin{cases} \dfrac{\mu x}{a+x^2} - D = 0, \\ r\left(1-\dfrac{x}{K}\right) - \dfrac{y}{a+x^2} = 0, \end{cases} \tag{2.1}$$

the first equation of (2.1) is equalized to

$$Dx^2 - \mu x + aD = 0. \tag{2.2}$$

Then, there are three cases about the solutions of Eq. (2.2).

Case 1. There is no positive solution for Eq. (2.2), if $\mu^2 - 4aD^2 < 0$.

Case 2. There is only one positive solution $x_2 = \frac{\mu}{2D}$ for Eq. (2.2), if $\mu^2 - 4aD^2 = 0$.
Thus, submitting x_2 into the first equation of (2.1) we have

$$y_2 = r(a+x_2^2)\left(1-\frac{\mu}{2KD}\right).$$

Therefore, when $K > \frac{\mu}{2D}$, $E_2(x_2, y_2)$ is unique positive equilibrium of model (1.2).

Case 3. There are two positive solutions

$$x_3 = \frac{\mu - \sqrt{\mu^2 - 4aD^2}}{2D}, \qquad x_4 = \frac{\mu + \sqrt{\mu^2 - 4aD^2}}{2D},$$

if $\mu^2 - 4aD^2 > 0$. Then, from the second equation of (2.1) we obtain

$$y_3 = r(a+x_3^2)\left(1-\frac{x_3}{K}\right), \qquad y_4 = r(a+x_4^2)\left(1-\frac{x_4}{K}\right).$$

Therefore, when $K > x_3$ or $K > x_4$, we obtain that $E_3(x_3, y_3)$ or $E_4(x_4, y_4)$ is the positive equilibria of model (1.2).