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Spatiotemporal dynamics of a Leslie–Gower predator–prey model incorporating a prey refuge

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ABSTRACT

In this paper, we investigate the spatiotemporal dynamics of a two-dimensional predator–prey model, which is based on a modified version of the Leslie–Gower scheme incorporating a prey refuge. We establish a Lyapunov function to prove the global stability of the equilibria with diffusion and determine the Turing space in the spatial domain. Furthermore, we perform a series of numerical simulations and find that the model dynamics exhibits complex Turing pattern replication: stripes, cold/hot spots-stripes coexistence and cold/hot spots patterns. The results indicate that the effect of the prey refuge for pattern formation is tremendous. This may enrich the dynamics of the effect of refuge on the predator-prey systems.

1. Introduction

A fundamental goal of theoretical ecology is to understand the interactions of individual organisms with each other and with the environment and to determine the distribution of populations and the structure of communities. An important type of interaction that effects population dynamics of all species is predation. Thus, predator–prey models have been in the focus of ecological science since the early days of this discipline. And continuous predator–prey models have been studied mathematically since the pioneering work of Lotka and Volterra, a simple ordinary differential system of interacting species that still bears their joint names. The principles of this model, conservation of mass and decomposition of the rates of change in birth and death processes, have remained valid until today and many theoretical ecologists adhere to these principles. In other words, the dynamic relationship between predators and their prey has long been and will continue to be one of dominant themes in both ecology and mathematical ecology due to its universal existence and importance [1–3].

In recent years, one of important predator–prey models is Leslie–Gower model [4,5], which has been extensively studied [6–22]. A modified version of Leslie–Gower predator–prey model with Holling-type II functional response takes the form

$$\frac{\mathrm{d}H}{\mathrm{d}t} = H(r - aH) - \frac{cHP}{b+H}, \qquad \frac{\mathrm{d}P}{\mathrm{d}t} = P\left(d - \frac{cP}{b+H}\right) \tag{1.1}$$

where H and P represent prey and predator population densities at time t, respectively. r, a, b, c, d are model parameters assuming only positive values. r is the growth rate of preys H. d describes the growth rate of predators P. a measures the strength of competition among individuals of species H. b measures the extent to which environment provides protection to prey H. c is the maximum value of the per capita reduction of H due to P.

We live in a spatial world, and the spatial component of ecological interactions has been identified as an important factor in how ecological communities are shaped. Understanding the role of space is challenging both theoretically and

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empirically [23]. Empirical evidence suggests that the spatial scale and structure of environment can influence population interactions [24]. In the research of pattern formation of spatial Lesile-Gower model, a major development was performed by Camara [25–27] with the following reaction–diffusion model:

$$\frac{\partial H}{\partial t} = H(r - aH) - \frac{cHP}{b+H} + D_1\nabla^2 H, \qquad \frac{\partial P}{\partial t} = P\left(d - \frac{cP}{b+H}\right) + D_2\nabla^2 P, \tag{1.2}$$

where D_1 and D_2 are the diffusion coefficients of prey and predator, respectively. $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the usual Laplacian operator in two-dimensional space.

In papers [25–27], the authors derived the conditions of Hopf and Turing bifurcation in the spatial domain and gave the numerical results of the pattern formation of the model. They found that the system exhibits spiral and chaos self-organization.

On the other hand, mite predator–prey interactions often exhibit spatial refugia, which afford the prey some degree of protection from predation and reduce the chance of extinction due to predation [9,16,28–42].

In fact, the effects of prey refuges on the population dynamics are very complex in nature, but for modeling purposes, it can be considered as constituted by two components: the first effects, which affect positively the growth of prey and negatively that of predators, comprise the reduction of prey mortality due to decrease in predation success. The second one may be the trade-offs and by-products of the hiding behavior of prey which could be advantageous or detrimental for all the interacting populations [31,35]. Kar [36] indicated that "increasing the amount of refuge can increase prey densities and lead to population outbreaks". Different from their declaration, Chen et al. [16] showed that the prey refuge has no influence on the persistence property of both predator and prey species, and the prey refuge could influence the densities of both prey and predator species greatly. Some work show that the equilibrium density of prey increases with prey refuges and its effect on that of predators is reverse when the number of prey hiding in refuges is large enough [29,35–37]. In [34,35,42], the authors obtained that the refuges, which protect a constant number of prey, have a stronger stabilizing effect on population dynamics than the refuges, which protect a constant proportion of prey. Ko and Ryu [39] studied a predator–prey model with Holling type II functional response incorporating a prey refuge under the homogeneous Neumann boundary condition, showed the existence and non-existence of non-constant positive steady-state solutions, investigated the asymptotic behavior of spatially inhomogeneous solutions and the local existence of periodic solutions.

Although there has been a great deal of researches on the effects of prey refuges on the population dynamics. But, to the best of our knowledge, little attention has been paid to study on the Turing pattern formation in the predation model incorporating a prey refuge.

Based on the above discussions, in this paper, we will focus on the spatiotemporal dynamics of a Leslie–Gower-type model incorporating a prey refuge. The rest of the paper is organized as follows: in the next section, we will establish the model and give some theorems about the stability property of the equilibria of the model, and determine the Turing space. In Section 3, we illustrate the emergence of Turing patterns via numerical simulations. Finally, conclusions and remarks are presented in Section 4.

2. The model and analysis

2.1. The model

In this paper, we will extend model (1.2) by incorporating a refuge protecting mH of the prey, where $m \in [0, 1)$ is constant. This leaves $(1 - m)H$ of the prey available to the predator, and modifying system (1.2) accordingly yields the system:

$$\begin{aligned} \frac{\partial H}{\partial t} &= H(r - aH) - \frac{(1-m)HP}{b+(1-m)H} + D_1\nabla^2 H, \\ \frac{\partial P}{\partial t} &= P\left(d - \frac{cP}{b+(1-m)H}\right) + D_2\nabla^2 P. \end{aligned} \tag{2.1}$$

The non-spatial model corresponding to (2.1) is as follows:

$$\begin{aligned} \frac{\mathrm{d}H}{\mathrm{d}t} &= H(r - aH) - \frac{(1-m)HP}{b+(1-m)H}, \\ \frac{\mathrm{d}P}{\mathrm{d}t} &= P\left(d - \frac{cP}{b+(1-m)H}\right). \end{aligned} \tag{2.2}$$

Model (2.1) is to be analyzed under the following non-zero initial conditions:

$$H(x, y, 0) > 0, \qquad P(x, y, 0) > 0, \quad (x, y) \in \Omega = [0, Lx] \times [0, Ly] \tag{2.3}$$