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Structural stability analysis of an algal bloom mathematical model in tropic interaction

Nurul Huda Gazi [a,*], Kalyan Das [b]

[a] St. Xavier's College, 30, Mother Teresa Sarani, Kolkata - 700016, India
[b] B.P. Poddar Institute of Management & Technology, 137, VIP Road, Kolkata-700052, India

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ABSTRACT

The paper deals with the dynamical behavior of plankton population ecosystem, mainly found in Sunderban mangrove area. The ecosystem is represented by a set of two dimensional non-linear differential equations involving zooplankton-phytoplankton population. Plankton populations undergo dramatic changes in marine ecology. We propose a description of plankton communities as excitable systems which resemble the behavior of excitable media. The delay parameter dependency of the various 'excitable' phenomena, trigger mechanism, threshold, and slow recovery, is clear, and permits ready investigation of the influence of properties of the physical environment, including variations in nutrient fluxes, temperature or population levels. We have analyzed the stability and bifurcation of the model system with and without delay. We have shown the existence and uniqueness of limit cycles in the rapid growth of the plankton population. We also studied the model system into a stochastic one, by incorporating random fluctuations of the environment. And we study the stochastic stability of the dynamical system in mean square sense around the interior equilibrium.

1. Introduction

The phenomena of plankton [1] communities in the occurrence of rapid explosions of population and almost equally rapid declines, are separated by periods of almost stationary high or low population levels. These events are of two types, 'Spring blooms' and 'red tides'. Both undergo dramatic surges of phytoplankton population, which grow by a plant-like photosynthesis processes. The outbursts of the processes are still not clearly understood. Some of the mechanisms proposed include the availability of trace elements such as iron [2] and vitamin B [3], the vertical stability of the water column [4] and salinity. The spring bloom is a sudden and strong bloom of phytoplankton such as diatoms or dinoflagellates in the spring in temperate and sub-polar bodies of water. In winter, the water is well mixed, i.e., water is circulated from the bottom to the top of the water column because the water is relatively colder and therefore maintains a more uniform density. In the early spring, the upper water layers have enough nutrients circulated up from bottom water to maintain phytoplankton growth but the phytoplankton are unable to grow as there is frequent mixing from wind and light levels are not yet strong enough. However, as the ocean warms in the later spring, the warm water will stay at the top of the water column as it is less dense. This will create a layer of stratification called the epilimnion in fresh water. At this time, the phytoplankton are maintained in water with enough light and abundant nutrients, allowing their population numbers to grow exponentially. In most cases the phytoplankton will use up the available nutrients in a matter of weeks or months, eventually dwindling their numbers in summer. Many species of diatom will sink to the bottom and create resting cysts when nutrient concentrations

* Corresponding author. Tel.: +91 33 3296 7065; fax: +91 33 2287 9966.
E-mail addresses: nursha@rediffmail.com (N.H. Gazi), kalyandas70@rediffmail.com (K. Das).

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run low. Also, it is not uncommon to see a succession of phytoplankton species reach their growth peaks at different times through the course of the bloom, as different species will have optimal nutrient uptake at different concentrations. An algal bloom or marine bloom or water bloom is a rapid increase in the population of algae in an aquatic system. Algal blooms may occur in freshwater as well as marine environments. Typically, only one or a few phytoplankton species are involved and some blooms may be recognized by discoloration of the water resulting from the high density of pigmented cells. Although there is no officially recognized threshold level, algae can be considered to be blooming at concentrations of hundreds to thousands of cells per milliliter, depending on the causative species. Algal bloom concentrations may reach millions of cells per milliliter. Colors observed are green, yellowish-brown, or red. Bright green blooms may also occur. These are the results of blue-green algae, which are actually bacteria (cyanobacteria).

Some algal blooms [1,4] are the result of an excess of nutrients (particularly phosphorus and nitrogen) into water, and higher concentrations of these nutrients in water cause increased growth of algae and green plants. As more algae and plants grow, others die. This dead organic matter becomes food for bacteria that decomposes it. With more food available, the bacteria increase in number and use up the dissolved oxygen in the water. When the dissolved oxygen content decreases, many fish and aquatic insects cannot survive. This results in a dead area. Algal blooms may also be of concern, as some species of algae produce neurotoxins. At the high cell concentrations reached during some blooms, these toxins may have severe biological impacts on wildlife. Algal blooms composed of phytoplankters, known to naturally produce biotoxins, are often called 'Harmful Algal Blooms', or 'HABs'. Algal blooms are monitored using biomass measurements coupled with the examination of species present. A widely-used measure of algal and cyanobacterial biomass is the chlorophyll concentration. Peak values of chlorophyll a for an oligotrophic lake are about 1–10 μg/l, while in a eutrophic lake they can reach 300 μg/l. In cases of hypereutrophy, such as the Hartbeespoort Dam in South Africa, maxima of chlorophyll can be as high as 3000 μg/l. So-called black water is a dark discoloration of sea water, first described in the Florida in January 2002. Algal blooms sometimes occur in drinking water supplies. In such cases, toxins from the bloom can survive standard water purifying treatments. Researchers at Florida International University in Miami are experimenting with 640 kHz ultrasound waves that create micro-pressure zones as hot as 3700 °C. This breaks some water molecules into reactive fragments that can kill algae. Algae are a large and diverse group of simple plant-like organisms, ranging from unicellular to multicellular forms. The largest and most complex marine forms are called seaweeds. They are considered 'plant-like' because of their photosynthetic ability, and 'simple' because they lack the distinct organs of higher such as leaves and vascular tissue. Though the prokaryotic Cyanobacteria (commonly referred to as Blue-green) were traditionally included as 'algae' in older textbooks, many modern sources regard this as outdated and restrict the term algae to eukaryotic organisms. All true algae therefore have a nucleus enclosed within a membrane and chloroplasts bound in one or more membranes. Algae constitute a paraphyletic and polyphyletic group: they do not represent a single evolutionary direction or line, but a level or grade of organization that may have developed several times in the early history of life on Earth.

Algae lack leaves, roots, and other organs that characterize higher plants. They are distinguished from protozoa in that they are photosynthetic. Many are photo-autotrophic, although there are some groups containing members that are mixotrophic, deriving energy both from photosynthesis and uptake of organic carbon either by osmotrophy, myzotrophy or phagotrophy. Some unicellular species rely entirely on external energy sources and have reduced or lost their photosynthetic apparatus. All algae have photosynthetic machinery ultimately derived from the cyanobacteria, and so produce oxygen as a byproduct of photosynthesis, unlike other photosynthetic bacteria such as purple and green sulfur bacteria [3,5]. Phytoplankton are the autotrophic component of plankton. Most phytoplankton are too small to be individually seen with the unaided. However, when present in high enough numbers, they may appear as a green discoloration of the water due to the presence of chlorophyll within their cells (although the actual color may vary with the species of phytoplankton present due to varying levels of chlorophyll or the presence of accessory pigments such as phycobiliproteins, xanthophylls, etc). Bacteria or Cyanophyta, is a phylum of bacteria that obtain their energy through photosynthesis. They are a significant component of the marine nitrogen cycle and an important primary producer in many areas of the ocean, but are also found on land. Stromatolites of fossilized oxygen-producing cyanobacteria have been found from 2.8 billion years ago. The ability of cyanobacteria to perform oxygenic photosynthesis is thought to have converted the early reducing atmosphere into an oxidizing one, which dramatically changed the life forms on Earth and provoked an explosion of bio-diversity. Chloroplasts in plants and eukaryotic algae have evolved from cyanobacteria. Many industrially important compounds, such as ammonia, nitric acid, organic nitrates (propellants and explosives), and cyanides, contain nitrogen. The very strong bond in elemental nitrogen dominates nitrogen chemistry, causing difficulty for both organisms and industry in converting the N_2 into useful compounds, and releasing large amounts of energy when these compounds burn or decay back into nitrogen gas.

'Red tide' [6,7,5,8] is a common name for a phenomenon known as an algal bloom, an event in which estuarine, marine, or fresh water algae accumulate rapidly in the water column, or 'bloom'. These algae, more specifically phytoplankton, are microscopic, single-celled protists, plant-like organisms that can form dense, visible patches near the water's surface. Certain species of phytoplankton contain photosynthetic pigments that vary in color from green to brown to red, and when the algae are present in high concentrations, the water appears to be discolored or murky, varying in color from white to almost black, normally being red or brown. Not all algal blooms are dense enough to cause water discoloration, and not all discolored waters associated with algal blooms are red. Additionally, red tides are not typically associated with tidal movement of water, hence the preference among scientists to use the term algal bloom. 'Red tide' is a colloquial term used to refer to a natural phenomenon known as a 'harmful algal bloom' or 'HAB'. Since (1) a wide variety of algal species can cause a red tide, (2) red tides are not necessarily red, and many have no discoloration at all, (3) are unrelated to movements of the tides, and