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A local existence result for an optimal control problem modeling the manoeuvring of an underwater vehicle

Francisco Periago [a], Jorge Tiago [b,*]

[a] Departamento de Matemática Aplicada y Estadística, ETSI Industriales, Universidad Politécnica de Cartagena, 30202 Cartagena, Spain

[b] Universidad de Castilla-La Mancha, ETSI Industriales, 13071 Ciudad Real, Spain

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ABSTRACT

We prove a local existence result for the manoeuvrability control of a submarine. The problem is formulated as an optimal control problem with a nonlinear and highly coupled system of ODEs for the state law, a Lagrange-type cost function, and nonlinear controls which take values on a convex and compact subset of $\mathbb{R}^3$. Finally, the existence of solution for this problem is obtained by applying a recent general existence result (see [P. Pedregal and J. Tiago, Existence results for optimal control problems with some special non-linear dependence on state and control, SIAM J. Control Optim. 48 (2) (2009) 415–437]) which, however, requires some modifications in order to be applicable in our specific case.

1. Introduction

In this paper we turn over the existence of solution for the model of manoeuvrability control of a submarine which has been recently proposed in [1] (see also [2–4]). It corresponds to a real-life engineering problem, so all the hypotheses and ingredients that we will consider in the sequel are motivated by real (non-academic) requirements. To describe such a model, a state vector is defined:

$$\mathbf{x} = (x, y, z, \phi, \theta, \psi, u, v, w, p, q, r) \in \Omega \subset \mathbb{R}^{12}, \tag{1}$$

where $X_{world} = (x, y, z; \phi, \theta, \psi)$ indicates the position and orientation of the submarine in the world fixed coordinate system, and $V_{body} = (u, v, w; p, q, r)$ is the vector of linear and angular velocities measured in the body coordinate system. Throughout this paper we follow the usual SNAME notation [2]. Permitted ranges of Euler angles are

$$-\pi < \phi < \pi, \qquad -\frac{\pi}{2} < \theta < \frac{\pi}{2}, \qquad 0 < \psi < 2\pi, \tag{2}$$

and so

$$\Omega = \mathbb{R}^3 \times]-\pi, \pi[\times \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[\times]-0, 2\pi[\times \mathbb{R}^6.$$

The control vector is

$$\mathbf{u} = (\delta_b, \delta_s, \delta_r), \tag{3}$$

* Corresponding author. Tel.: +34 926295300; fax: +34 926295361.
E-mail addresses: f.periago@upct.es (F. Periago), jorge.tiago@uclm.es, jfdtiago@gmail.com (J. Tiago).

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where δ_b and δ_s represent, respectively, the angles of the bow and stern coupled planes, and δ_r is the deflection of the rudder. These controls act on the system in linear and quadratic forms. Therefore, it is convenient to consider the mapping

$$\Phi(\mathbf{u}) = \left(\mathbf{u}, \mathbf{u}^2\right) \equiv \left(\delta_b, \delta_s, \delta_r, \delta_b^2, \delta_s^2, \delta_r^2\right) \in \mathbb{R}^6.$$

Admissible controls $\mathbf{u}$ are measurable functions that should lie in a certain set $K \subset \mathbb{R}^3$, which, in our case, is given by

$$K = [-a_1, a_1] \times [-a_2, a_2] \times [-a_3, a_3],$$

with $0 < a_1, a_2, a_3 < \pi/2$. Finally, the state law is described by a system of twelve ordinary differential equations

$$\mathbf{x}'(t) = Q(\mathbf{x}(t))\,\Phi(\mathbf{u}(t)) + Q_0(\mathbf{x}(t)) \tag{4}$$

where

$$Q : \mathbb{R}^{12} \to \mathcal{M}^{12\times 6} \quad \text{and} \quad Q_0 : \mathbb{R}^{12} \to \mathbb{R}^{12}$$

will be described in Section 3. At this point, we just indicate that the right-hand side of (4) includes both kinematic and dynamic equations of motion (see [1–4] for more details).

The manoeuvrability control problem for an underwater vehicle relates to a situation where we want to reach (or to be very close to) a final state $\mathbf{x}^{\mathrm{T}}$ in time T, while minimizing the use of control during the time interval $[0, T]$. Typically, this type of problem is formulated as an exact or approximate controllability problem. However, in many real situations, there is no need for *all* the components of the state variable to be close to a final target $\mathbf{x}^{\mathrm{T}}$ at time T, but only *some of them* (see [1] for some examples). To the knowledge of the authors, there are no satisfactory results in the literature for these nonlinear controllability problems in which controls appear in a nonlinear form. Notice also that if a controllability problem has a solution, then such a solution may be obtained by using an optimal control formulation as we propose below.

The requirement for the minimum use of control can be understood as minimizing the typical cost

$$\int_0^{\mathrm{T}} \|\mathbf{u}(t)\|^2\,\mathrm{d}t \tag{5}$$

while the previous aspect of the final state $\mathbf{x}^{\mathrm{T}}$ can be seen as minimizing

$$\begin{aligned}\frac{1}{2}\left\|\mathbf{x}(T) - \mathbf{x}^{\mathrm{T}}\right\|^2 &= \frac{1}{2}\int_0^{\mathrm{T}} \frac{\mathrm{d}}{\mathrm{d}t}\left\|\mathbf{x}(t) - \mathbf{x}^{\mathrm{T}}\right\|^2 \mathrm{d}t + \frac{1}{2}\left\|\mathbf{x}(0) - \mathbf{x}^{\mathrm{T}}\right\|^2 \\ &= \int_0^{\mathrm{T}} \left\langle \mathbf{x}(t) - \mathbf{x}^{\mathrm{T}},\ Q(\mathbf{x}(t))\,\Phi(\mathbf{u}(t)) + Q_0(\mathbf{x}(t))\right\rangle \mathrm{d}t + \frac{1}{2}\left\|\mathbf{x}(0) - \mathbf{x}^{\mathrm{T}}\right\|^2.\end{aligned}$$

Hence, we consider the cost

$$\int_0^{\mathrm{T}} \left[\left\langle \mathbf{x}(t) - \mathbf{x}^{\mathrm{T}},\ Q(\mathbf{x}(t))\,\Phi(\mathbf{u}(t)) + Q_0(\mathbf{x}(t))\right\rangle + \|\mathbf{u}(t)\|^2\right]\mathrm{d}t = \int_0^{\mathrm{T}} \left[c(\mathbf{x}(t))\,\Phi(\mathbf{u}(t)) + c_0(\mathbf{x}(t))\right]\mathrm{d}t$$

where the vector c is given by

$$\begin{cases} c_i(\mathbf{x}) = \displaystyle\sum_{j=1}^{12} \left(\mathbf{x} - \mathbf{x}^{\mathrm{T}}\right)_j Q_{ji}, & i = 1, 2, 3, \\ c_i(\mathbf{x}) = \displaystyle\sum_{j=1}^{12} \left(\mathbf{x} - \mathbf{x}^{\mathrm{T}}\right)_j Q_{ji} + 1, & i = 4, 5, 6, \end{cases}$$

and

$$c_0(\mathbf{x}) = \left\langle \mathbf{x} - \mathbf{x}^{\mathrm{T}},\ Q_0(\mathbf{x})\right\rangle.$$

Notice that the above expression for vector c is a particular case of a more general expression

$$\begin{cases} \hat{c}_i(\mathbf{x}) = \displaystyle\sum_{j=1}^{12} \alpha_j \left(\mathbf{x} - \mathbf{x}^{\mathrm{T}}\right)_j Q_{ji}, & i = 1, 2, 3, \\ \hat{c}_i(\mathbf{x}) = \displaystyle\sum_{j=1}^{12} \alpha_j \left(\mathbf{x} - \mathbf{x}^{\mathrm{T}}\right)_j Q_{ji} + \beta_i, & i = 4, 5, 6, \end{cases}$$

where $\alpha_j \geq 0$, $1 \leq j \leq 12$, and $\beta_i \geq 0$, $4 \leq i \leq 6$, correspond to some penalty parameters which are introduced to weigh at convenience (5) and the fact that some components of the state vector are close to a final given target at time T. For some interesting (from the practical point of view) choices of the penalty parameters, we refer the reader to [1]. For simplicity and since it does not change the problem mathematically, in the sequel we have taken these parameters equal to 1.