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Periodic solutions for an equation governing dynamics of a renewable resource subjected to Allee effects

Seshadev Padhi [a], P.D.N. Srinivasu [b,*], G. Kiran Kumar [b]

[a] Department of Applied Mathematics, Birla Institute of Technology, Mesra, Ranchi, India

[b] Department of Mathematics, Andhra University, Visakhapatnam, India

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ABSTRACT

In this article the minimum number of positive periodic solutions admitted by a non-autonomous scalar differential equation is estimated. This result is employed to find the minimum number of positive periodic solutions admitted by a model representing dynamics of a renewable resource that is subjected to Allee effects in a seasonally varying environment. The Allee effect refers to a decrease in population growth rate at low population densities. Leggett–Williams multiple fixed point theorem is used to establish the existence of positive periodic solutions.

1. Introduction

Periodicity plays an important role in the problems associated with real world applications, in particular, issues related to eco-system dynamics. There has been considerable contribution in recent years on the existence of periodic and almost periodic solutions of differential equations having periodic causal functions. In this article we are interested in investigating the existence of multiple periodic solutions of a first order differential equation representing growth of a renewable resource that is subjected to Allee effects in a seasonally varying environment.

Many authors have used fixed point theorems involving cone expansion and cone compression methods, upper–lower solution methods and iterative techniques to find existence of either one or multiple positive periodic solutions of various differential equations, for instance see [1–6]. In a recent work Padhi and Srivastava [7] have obtained sufficient conditions for the existence of at least three positive periodic solutions of a functional differential equation using Leggett–Williams multiple fixed point theorem [8]. Motivated by the results of [7], in this paper we have made an attempt to use Leggett–Williams multiple fixed point theorem [8] to establish the existence of at least two positive periodic solutions for the considered model.

Allee effects have become much studied in recent years, largely because of their potential role in extinctions of already endangered, rare or dramatically declining species [9–11]. The Allee effect refers to a decrease in population growth rate at low population densities [12,13,10,14–18]. There are several mechanisms that create Allee effects in populations. A classification of these effects are presented in [19]. A few real world examples exhibiting these Allee effects can be found in [20,21,14,22–25]. A critical review on single species models subjected to Allee effects is presented in [26]. An equation representing growth of a species with Allee effects and the associated dynamics are discussed in [27,28]. Mathematical component of most of the available literature deals with differential equations with constant coefficients. Although seasonality is known to have considerable impact on the species dynamics, to our knowledge there does not exist any literature that discusses the dynamics of a renewable resource subjected to Allee effects in a seasonally varying environment.

* Corresponding author. Tel.: +91 8912844703.
E-mail addresses: ses_2312@yahoo.co.in (S. Padhi), pdns@rediffmail.com (P.D.N. Srinivasu), gkkumaraa@gmail.com (G.K. Kumar).

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In this article, we introduce seasonality into the resource dynamic equation by assuming the involved coefficients to be periodic as in [29]. Our interest is to find an estimate on the number of positive periodic solutions admitted by the considered model.

Section wise division of the article is as follows. In the next section we discuss about the model. In Section 3 the existence of multiple positive periodic solutions for a fairly general scalar differential equation is established. Existence of at least two positive periodic solutions for the resource dynamic equation is proved in Section 4 through an application of the general result obtained in Section 3. Section 5 illustrates the existence result through an example followed by discussion in Section 6.

2. The model

Let us consider the following equation representing dynamics of a renewable resource y, that is subjected to Allee effects [27,28]:

$$\frac{dy}{dt} = ay(y-b)(c-y), \quad a>0, 0<b<c \tag{1}$$

where the constants a, c and b represent respectively intrinsic growth rate, carrying capacity of the resource and the threshold value below which the growth rate of the resource is negative. It is well known that Eq. (1) admits two positive solutions given by $y(t)=b$ and $y(t)=c$ and one trivial solution as its equilibrium solutions. Eq. (1) can be non-dimensionalized to reduce the number of parameters to obtain

$$\frac{dy}{dt} = y(y-\beta)(1-y), \quad 0<\beta<1. \tag{2}$$

Since we are interested in the dynamics of a renewable resource in a seasonally varying environment we assume the coefficients a, b, c to be nonnegative T-periodic functions of same period and study the existence of T-periodic solutions. Thus we consider

$$\frac{dy}{dt} = a(t)y(y-b(t))(c(t)-y) \tag{3}$$

where the nonnegative functions $c(t)$ and $b(t)$ stand for seasonal dependent carrying capacity and threshold function of the species respectively satisfying

$$0<b(t)<c(t) \tag{4}$$

and $a(t)$ represents time dependent intrinsic growth rate of the resource. Clearly, we have the trivial solution ($y(t)\equiv 0$) to be a periodic solution of (3). Since the study deals with resource dynamics we are interested in the existence of positive periodic solutions of the considered equation.

Considering the transformation

$$y(t)=c(t)x(t) \tag{5}$$

Eq. (3) gets transformed to

$$\frac{dx}{dt} = -\left(a(t)c^2(t)k(t)+\frac{c'(t)}{c(t)}\right)x + a(t)c^2(t)\left((1+k(t))-x\right)x^2 \tag{6}$$

where

$$k(t)=\frac{b(t)}{c(t)}<1. \tag{7}$$

Note that (6) is a particular case of a general scalar differential equation of the form

$$\frac{dx}{dt} = -A(t)x(t)+f(t,x(t)) \tag{8}$$

where $A\in C(R,R), f\in C(R\times R,R)$ satisfying $A(t+T)=A(t), f(t+T,x)=f(t,x)$.

In the next section we shall establish existence of positive periodic solutions for Eq. (8) under reasonable assumptions on the causal functions and apply these existence results to Eq. (6) to obtain information on existence of positive T-periodic solutions.

3. Existence of periodic solutions

In this section, we establish the existence of at least two positive periodic solutions for (8) using Leggett–Williams multiple fixed point theorem [8] on cones. The following lemma is easily verifiable.