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A new smoothing Broyden-like method for solving the mixed complementarity problem with a P_0-function☆

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ABSTRACT

A new smoothing Broyden-like algorithm for solving the mixed complementarity problems (denoted by MCP) is proposed in this paper. The algorithm considered here first uses a floating genetic algorithm to quickly calculate superior results, which are close to the precise solution of the equivalent optimization problems for the mixed complementarity problems, and then by taking the results as the initial values in the following smoothing Broyden-like algorithm, which is based on a perturbed mid function and makes use of the line search rule of D.-H. Li (2000) [16], we obtain a satisfactory approximate solution. In addition, the existence and continuity of a smooth path for solving the mixed complementarity problem with a P_0-function are also discussed. The new smoothing Broyden-like algorithm combines the advantages of quasi-Newton method of local superlinear convergence and a floating genetic algorithm in group search, and global convergence. Numerical results show that the new method is feasible and effective.

1. Introduction

Consider the following mixed complementarity problem (denoted by MCP(F): to find a vector $x \in X \subset R^n$ such that

$$F(x)^{\mathrm{T}}(y-x) \geq 0, \quad \forall y \in X \tag{1.1}$$

where

$$X = \prod_{i=1}^{n}[l_i, u_i], \quad -\infty \leq l_i < u_i \leq +\infty, \; i = 1, 2, \ldots, n.$$

In what follows, we suppose that F be a continuous differentiable P_0-function. A function $F : R^n \to R^n$ is said to be a P_0-function if, for all $u, v \in R^n$ with $u \neq v$, there exists an index i such that

$$u_i \neq v_i \quad \text{and} \quad (u_i - v_i)[F_i(u) - F_i(v)] \geq 0.$$

Furthermore, for the same index i, if the relation

$$u_i \neq v_i \quad \text{and} \quad (u_i - v_i)[F_i(u) - F_i(v)] > 0$$

holds, then the function $F : R^n \to R^n$ is said to be a P-function.

Due to the importance in both theory and applications (Refs. [1–3]), the numerical solution of MCP(F) has raised much interest among researchers (Refs. [4–11,21]). On the basis of the ideas of smoothing Newton methods, we propose in this

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paper a new smoothing function which is obtained by smoothing a perturbed mid function. The new smoothing function has an important property (see Lemma 2.5), not satisfied by many other smoothing functions. This unique feature allows us to study the boundedness of the iteration sequence generated by the smoothing methods for solving MCP(F) with a P_0-function under the assumption (4.1). Then, a smoothing Broyden-like method is proposed.

However, it is well-known that quasi-Newton method is locally convergent, depending greatly on its initial point. Hence, there is increasing focus on how to design an efficient method with global convergence.

Recently, the genetic algorithm has become a hot topic in the research field of optimization problems and has raised much interest among researchers (Refs. [12–15]). By simulating natural selection and genetic systems, this method is marked by its prominent excellence in global search and the high speed of initial search. Hence, it can be widely used in addressing large-scale and complex optimization problems and nonlinear equations with a changeable state. However, this method has its own limitations, such as the inferior search results in the later phases. Furthermore, the final results are often hardly in accord with the required precision.

The algorithm considered in this paper first uses a floating genetic algorithm to quickly calculate superior results, which are close to the precise solution of the equivalent optimization problems for the mixed complementarity problems, and then by taking the results as the initial values for the following smoothing Broyden-like algorithm, which is based on a perturbed mid function and makes use of the line search rule of Li from Ref. [16], we obtain a satisfactory approximate solution. In addition, the existence and continuity of a smooth path for solving the mixed complementarity problem with a P_0-function are also discussed. The new smoothing Broyden-like algorithm combines the advantages of quasi-Newton method in local superlinear convergence and a floating genetic algorithm in group search, and global convergence; thereby, it can solve the problem of quasi Newton method as regards the choice of starting point.

The paper is organized as follows. In Section 2, we introduce a new smoothing function and study a few properties of the smoothing function. The boundedness of the iteration sequence under assumption (1.1) is also investigated. In Section 3, we present a new smoothing Broyden-like algorithm. In Section 4, we discuss the global convergence and local superlinear convergence of the algorithm. Some numerical results are reported in Section 5 and the conclusion is given in Section 6.

In our notation, all vectors are column vectors, ^{T} denotes the transpose, R^n denotes the space of n-dimensional real column vectors, and R^n_+ [resp., R^n_{++}] denotes the nonnegative [resp., positive] orthant in R^n. We define $N := \{1, 2, \ldots, n\}$. For any vector $u \in R^n$, we denote by $\mathrm{diag}\{u_i : i \in N\}$ the diagonal matrix whose ith diagonal element is u_i and $\mathrm{vec}\{u_i : i \in N\}$ the vector u. For any continuously differentiable function $F = (F_1, F_2, \ldots, F_n)^T : R^m \to R^m$, we denote its Jacobian by $F' = (\nabla F_1, \nabla F_2, \ldots, \nabla F_n)^T$, where ∇F_i denotes the gradient of F_i for $i = 1, 2, \ldots, m$.

2. The smoothing function and its properties

For any $(a, b, c) \in R^3$, the mid function is defined by

$$\bar{\phi}(a, b, c) := \mathrm{mid}\{a, b, c\} = \begin{cases} a, & \text{if } c \le a \le b, \\ b, & \text{if } c \le b \le a, \\ c, & \text{if } b \le c \le a. \end{cases}$$

Since $\bar{\phi}(\cdot, \cdot, \cdot)$ is nonsmooth, to apply a Newton-type method to solve MCP(F), we introduce a parameter $\mu \in R$ into the mid function, and construct the following function:

$$\hat{\phi}(\mu, a, b, c) := \mathrm{mid}\{a, b + \mu^2(a + c), c\}; \tag{2.1}$$

then, by smoothing $\hat{\phi}(\mu, a, b, c)$, we can obtain the smoothing MCP function as follows:

$$\phi(\mu, a, b, c) = (a + c) - \sqrt{[(a - b) - \mu^2(a + c)]^2 + \mu^2} + \sqrt{[(c - b) - \mu^2(a + c)]^2 + \mu^2} \tag{2.2}$$

One can easily see that

$$\phi(0, a, b, c) = \hat{\phi}(0, a, b, c) = \bar{\phi}(a, b, c).$$

The following lemma gives two simple properties of the smoothing function $\phi(\cdot, \cdot, \cdot, \cdot)$ defined by (2.2). Its proof is obvious.

Lemma 2.1. *Let $(\mu, a, b, c) \in R^4$ and $\phi(\mu, a, b, c)$ be defined by (2.2). Then,*

(1) $\phi(0, a, b, c) = 0,\ a > c \Leftrightarrow \mathrm{mid}\{a, b, c\} = 0,\ a > c$;
(2) *$\phi(\mu, a, b, c)$ is continuously differentiable for all points in R^4 different from $(0, a, a, c)$ and $(0, a, c, c)$ for arbitrary $a, c \in R$. In particular, if $\mu > 0$, $\phi(\mu, a, b, c)$ is continuously differentiable for arbitrary $(a, b, c) \in R^3$.*

For the sake of simplicity, we use the following notation:

$$z := (\mu, x, y) \in R_{++} \times R^{2n}, \tag{2.3a}$$

$$H(z) := \begin{bmatrix} e^{\mu} - 1 \\ F(z) \end{bmatrix} = \begin{bmatrix} e^{\mu} - 1 \\ y - F(x) \\ \Phi(z) \end{bmatrix} \tag{2.3b}$$