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Unsteady flow with heat and mass transfer of a third grade fluid over a stretching surface in the presence of chemical reaction

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ABSTRACT

This paper describes the unsteady flow with heat and mass transfer characteristics in a third grade fluid bounded by a stretching sheet. The resulting problems are solved by means of homotopy analysis method (HAM). Convergence of derived series solutions is explicitly discussed. Graphical results for various interesting parameters are presented and analyzed.
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1. Introduction

Recently, the flows of non-Newtonian fluids [1–10] have attracted the attention of several researchers. Due to the variety of non-Newtonian fluids in nature, the flows of such fluids cannot be described by a single constitutive relationship between stress and rate of strain. Examples include hot rolling, drilling muds, oil and greases, clay coating and suspensions, paper products and many others. The resulting equations are of higher order and more complicated than the Navier–Stokes equations [11,12]. Therefore, it is not an easy task to develop either a numerical or analytic solution of the resulting equations in non-Newtonian fluids.

In the past, much attention has been given to the time-independent flows induced by a stretching surface. Little efforts are devoted to examine time-dependent stretching flow problems. In recent times, Sajid et al. [13] considered the unsteady flow and heat transfer of a second grade fluid over a stretching sheet. The purpose of the present investigation is two fold. Firstly, to extend the analysis of Ref. [13] from second grade fluid to third grade fluid. Secondly, to consider heat and mass transfer characteristics in the absence of viscous dissipation and heat generation. The paper is arranged as follows. The formulation of the problem is described in Section 2. In Section 3, the series solutions of velocity, temperature and concentration are derived using homotopy analysis method (HAM) [14–30]. Convergence of the solution is presented in Section 4. Section 5 deals with the discussion of graphs and tables. Special emphasis has been given to the various embedding parameters. The skin friction coefficients, local Nusselt number and the local Sherwood number are also analyzed. Finally, the concluding remarks are given in Section 6.

2. Mathematical formulation

Let us choose a Cartesian coordinate system. We consider the unsteady flow of a third grade fluid over a stretching surface. The heat and mass transfer in the presence of a chemical reaction are taken into account. The boundary layer equations

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governing the flow are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\begin{aligned}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y} &= \nu\frac{\partial^2 u}{\partial y^2}+\frac{\alpha_1}{\rho}\left(\frac{\partial^3 u}{\partial y^2\partial t}+u\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}+3\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial^3 u}{\partial y^3}\right)\\ &\quad+\frac{2\alpha_2}{\rho}\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+\frac{6\beta_3}{\rho}\left(\left(\frac{\partial u}{\partial y}\right)^2\frac{\partial^2 u}{\partial y^2}\right),\end{aligned} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}. \tag{3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-K_1C. \tag{4}$$

Here, u and v are the velocities in the x, y directions, respectively, T is the temperature, C is the concentration, ρ is the fluid density, ν is the kinematic viscosity, α_i $(i=1,2)$ and β_3 are the material constants, α is the thermal diffusivity and D is the diffusion coefficient of the diffusing species. Note that the viscous dissipation effects in the energy equation are neglected.

The subjected boundary conditions are

$$\begin{aligned}&u=u_w(x)=ax, \qquad v=0, \qquad T=T_w, \qquad C=C_w \quad \text{at } y=0,\\ &u\to 0, \qquad v\to 0, \qquad T\to T_\infty, \qquad C\to C_\infty \quad \text{as } y\to\infty,\end{aligned} \tag{5}$$

where the constants a and b are positive. Upon making use of the following non-dimensional variables

$$\begin{aligned}&\eta=\sqrt{\frac{a}{\nu\xi}}y, \qquad u=axf'(\eta,\xi), \qquad v=-\sqrt{a\nu\xi}f(\eta,\xi),\\ &\xi=1-\exp[-\tau], \qquad \tau=at, \qquad \theta(\eta,\xi)=\frac{T-T_\infty}{T_w-T_\infty}, \qquad \varphi(\eta,\xi)=\frac{C-C_\infty}{C_w-C_\infty}.\end{aligned} \tag{6}$$

Eq. (1) is identically satisfied and Eqs. (2)–(5) can be written as

$$\begin{aligned}&\{\xi-\epsilon_1(1-\xi)\}f'''-\xi^2(f'^2-ff'')+\xi(1-\xi)\left(\frac{\eta}{2}f''-\xi\frac{\partial f'}{\partial\xi}\right)\\ &\quad+\epsilon_1\left(\begin{array}{c}\xi f'f'''-\xi ff^{\text{iv}}-\\ \frac{\eta}{2}(1-\xi)f^{\text{iv}}+\xi(1-\xi)\frac{\partial f'''}{\partial\xi}\end{array}\right)+(3\epsilon_1+2\epsilon_2)\xi f''^2+6\phi\phi_1 f''^2f'''=0,\end{aligned} \tag{7}$$

$$\theta''+\Pr(1-\xi)\left(\frac{\eta}{2}\theta'-\xi\frac{\partial\theta}{\partial\xi}\right)+\Pr\xi f\theta'=0, \tag{8}$$

$$\varphi''+Sc(1-\xi)\left(\frac{\eta}{2}\varphi'-\xi\frac{\partial\varphi}{\partial\xi}\right)+Sc\xi f\varphi'-Sc\xi\gamma\varphi=0, \tag{9}$$

$$\begin{aligned}&f(0,\xi)=0, \qquad f'(0,\xi)=1, \qquad \theta(0,\xi)=1, \qquad \varphi(0,\xi)=1,\\ &f'(\infty,\xi)=0, \qquad f''(\infty,\xi)=0, \qquad \theta(\infty,\xi)=0, \qquad \varphi(\infty,\xi)=0,\end{aligned} \tag{10}$$

where

$$\begin{aligned}&\epsilon_1=\frac{a\alpha_1}{\mu}, \qquad \epsilon_2=\frac{a\alpha_2}{\mu}, \qquad \phi=\frac{\beta_3}{\mu}, \qquad \phi_1=\frac{ax^2}{\upsilon}, \qquad \Pr=\frac{\upsilon}{\alpha},\\ &Sc=\frac{\upsilon}{D}, \qquad \gamma=\frac{K_1}{a}\end{aligned} \tag{11}$$

and μ is the dynamic viscosity.

The local Nusselt Nu_x and local Sherwood Sh numbers are defined as

$$Nu_x=-\frac{x\left(\frac{\partial T}{\partial y}\right)_{y=0}}{(T_w-T_\infty)}, \qquad Sh=-\frac{x\left(\frac{\partial C}{\partial y}\right)_{y=0}}{(C_w-C_\infty)}. \tag{12}$$

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