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New sufficient conditions for global asymptotic stability of Cohen–Grossberg neural networks with time-varying delays

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ABSTRACT

In this paper, a class of Cohen–Grossberg neural networks with time-varying delays are considered. Without assuming the boundedness and monotonicity of activation functions, we establish new sufficient conditions for the existence, uniqueness and global asymptotic stability of the equilibrium point for such delayed Cohen–Grossberg neural networks. Numerical examples are provided to show that the proposed criteria are less conservative than some results in the literature.

1. Introduction

Over the past decades, different classes of neural networks have attracted the attention of researchers, due to their wide range of applications in classification, parallel computing, associate memory and optimization problems (see [1–8,10–18] and the references therein). Among them the Cohen–Grossberg neural networks have been an active research topic (see [1–3, 6,7,10–12,14–17]), since it could include a lot of models from evolutionary theory, population biology and neurobiology [3]. Under the assumption that the activation functions are bounded, some sufficient conditions about the global asymptotic stability of delayed Cohen–Grossberg neural networks have already been obtained by some authors, such as [2,6,16] and the references therein. The aim of this paper is to provide some new results on delayed Cohen–Grossberg neural networks, without assuming the boundedness and monotonicity of activation functions.

2. Model description and preliminaries

Cohen–Grossberg neural networks model with time-varying delay is described by the following differential equation:

$$\dot{x}_i(t) = d_i(x_i(t)) \left[-c_i(x_i(t)) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t - \tau_j(t))) + u_i \right], \quad i = 1, 2, \ldots, n, \tag{1}$$

where n denotes the number of neurons, x_i denotes the state of the ith neuron, $d_i(x_i)$ represents an amplification function, and $c_i(x_i)$ is a behaved function. The constants a_{ij} denote the strengths of the neuron interconnections within the network, and the constants b_{ij} denote the strengths of the neuron interconnections with time delays. The functions $f_i(\cdot)$ denote neuronal activations and the constants u_i are some external inputs. Time delays $\tau_i(t)$ are bounded with $0 \le \tau_i(t) \le \tau$, $i = 1, 2, \ldots, n$.

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Accompanying the neural system (1) is an initial condition: $x_i(t) = \zeta_i(t)$, $t \in [-\tau, 0]$, where $\zeta_i(t)$ is a continuous function from $[-\tau, 0]$ to $\mathbb{R}$.

System (1) can also be expressed in the following vector–matrix form:

$$\dot{x}(t) = D(x(t))\left[-C(x(t)) + Af(x(t)) + Bf(x(t-\tau(t))) + u\right], \tag{2}$$

where

$$\begin{aligned}
&x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^{\mathrm{T}},\\
&D(x(t)) = \operatorname{diag}\left(d_1(x_1(t)), d_2(x_2(t)), \ldots, d_n(x_n(t))\right),\\
&C(x(t)) = \left(c_1(x_1(t)), c_2(x_2(t)), \ldots, c_n(x_n(t))\right)^{\mathrm{T}},\\
&A = (a_{ij})_{n\times n}, \qquad B = (b_{ij})_{n\times n},\\
&u = (u_1, u_2, \ldots, u_n)^{\mathrm{T}},\\
&f(x(t-\tau(t))) = \left(f_1(x_1(t-\tau_1(t))), f_2(x_2(t-\tau_2(t))), \ldots, f_n(x_n(t-\tau_n(t)))\right)^{\mathrm{T}}.
\end{aligned}$$

We give the following assumptions:

A_1: The functions $d_i(\xi)$, $i = 1, 2, \ldots, n$, are continuously bounded, and there exist positive constants η_i and ρ_i, such that $0 < \eta_i \le d_i(\xi) \le \rho_i$, $\forall \xi \in \mathbb{R}$.

A_2: The functions c_i are continuous and there exist constants $\gamma_i > 0$ such that

$$\frac{c_i(\xi_1) - c_i(\xi_2)}{\xi_1 - \xi_2} \ge \gamma_i > 0, \quad \forall \xi_1, \xi_2 \in \mathbb{R}, \xi_1 \ne \xi_2.$$

A_3: The activation functions $f_i(\cdot)$ are Lipschitz continuous, i.e., there exist constants $k_i > 0$ such that

$$|f_i(\xi_1) - f_i(\xi_2)| \le k_i\,|\xi_1 - \xi_2|\,, \quad i = 1, 2, \ldots, n, \forall \xi_1, \xi_2 \in \mathbb{R}.$$

For convenience, we introduce some notations.

$$K = \operatorname{diag}\{k_1, k_2, \ldots, k_n\}, \qquad \Gamma = \operatorname{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_n\}.$$

For any vector $v = (v_1, v_2, \ldots, v_n)^{\mathrm{T}} \in \mathbb{R}^n$, $\|v\|$ denotes a vector norm. The three commonly used vector norms are

$$\|v\|_1 = \sum_{i=1}^{n} |v_i|\,, \qquad \|v\|_2 = \left(\sum_{i=1}^{n} |v_i|^2\right)^{\frac{1}{2}}, \qquad \|v\|_\infty = \max_{1\le i\le n} |v_i|\,.$$

For any matrix $Q = (q_{ij})_{n\times n}$,

$$\|Q\|_1 = \max_{1\le i\le n}\sum_{j=1}^{n} |q_{ji}|\,, \qquad \|Q\|_2 = \left(\lambda_{\max}(Q^{\mathrm{T}}Q)\right)^{\frac{1}{2}},$$

where $\lambda_{\max}(Q^{\mathrm{T}}Q)$ denotes the maximum eigenvalue of matrix $Q^{\mathrm{T}}Q$.

Lemma 1 (*Forti and Tesi, [8]*). *If $H : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function and satisfies the following conditions:*

(i) *$H(x) \ne H(y)$ for all $x \ne y$,*
(ii) *$\|H(x)\| \to \infty$ as $\|x\| \to \infty$,*

then, $H(x)$ is homeomorphism of $\mathbb{R}^n$.

Lemma 2. *For any $x, y \in \mathbb{R}^n$ and positive definite matrix $Q \in \mathbb{R}^{n\times n}$, the following matrix inequality holds:*

$$2x^{\mathrm{T}}y \le x^{\mathrm{T}}Qx + y^{\mathrm{T}}Q^{-1}y.$$

3. Existence and uniqueness analysis of equilibrium point

In this section, we present sufficient conditions for the existence and uniqueness of the equilibrium point for system (1).

Theorem 1. *Under the assumptions A_1–A_3, the system (2) has a unique equilibrium point for every input u if there exist three positive diagonal matrices $P = \operatorname{diag}\{p_1, p_2, \ldots, p_n\}$, $S = \operatorname{diag}\{s_1, s_2, \ldots, s_n\}$, and $W = \operatorname{diag}\{w_1, w_2, \ldots, w_n\}$, such that*

$$\kappa_i \triangleq 2w_i\gamma_i - (p_i + s_i)k_i^2 - \left\|WAP^{-\frac{1}{2}}\right\|_2^2 - \left\|WBS^{-\frac{1}{2}}\right\|_2^2 > 0, \quad \forall i = 1, 2, \ldots, n. \tag{3}$$