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Stability of bifurcating periodic solutions for a single delayed inertial neuron model under periodic excitation

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ABSTRACT

Using the averaged method, we investigate the dynamical characteristics of a single inertial neuron model with time delay under periodic external stimuli. It is shown that the system will lose its stability when the time delay is increased and will give rise to a quasi-periodic motion and chaos under the interaction of the periodic excitation. Numerical simulations show that the results of the analytical method are correct by a comparison with those of direct numerical integration.

1. Introduction

In recent years, dynamical characteristics of neural networks have become the subject of intense research. This is because evidence from the experimental and theoretical studies indicate that a mammal's brain may be exploiting dynamic attractors for its encoding and storage of associative memories rather than static attractor, as supposed in most studies of neural network [1–3,26]. There are also some biological background for the inclusion of an inductance term in a neural system. For example, the membrane of a hair cell in the semicircular canals of some animals can be described by equivalent circuits that contain an inductance [4,5], and the inertia can be considered an effective tool to help in the generation of chaos in neural systems. Moreover, it is very useful and significant to introduce an inertial term(the influence of inductance) into the standard neural system. In reality, time delays often occur in many systems due to the finite switching speed of amplifies in electronic neural networks, or due to the finite signal propagation time in biological networks [1–3,19–22,24]. Since the discrete delay usually discrete that the present state of the system changes at time t in a manner affected by the value of some variables at time $t-\tau$. Many authors investigated the bifurcation and chaotic phenomena for the different models with time delays [6–10]. However, most of the existing literature on theoretical studies of artificial neural networks are predominantly concerned with autonomous systems [6–10]. At the same time, almost all the works for the mentioned-above papers are based on applying the normal form and center manifold theorem introduced by Hassard et al. [12] to determine the direction of Hopf bifurcation and stability of bifurcating periodic solutions. Based on the frequency domain approach, some authors have analyzed the associated characteristic equation to obtain stability for the bifurcating periodic solutions [13]. However, external stimuli plays a very important role in some important applications of neural networks, for example, the moving image processing by using cellular neural networks. It is also needed when neural networks are designed for solving optimization problems [14,15]. More recently, researchers have obtained sufficient conditions for the existence of a globally attractive periodic solution by adding a given periodic external stimulus to standard Hopfield model with time delays [16]. But the stability of bifurcating periodic solution was not involved.

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In this paper, one of the simplest but widely used model, the delayed inertial neuron model [11,17,23,25], is considered by introducing a periodic external stimuli. We study how a temporally varying, especially, a periodic stimuli, can influence the dynamics of a single inertial neuron model. The mathematical model is given as:

$$\ddot{x} = -a\dot{x} - bx + c\tanh[x(t-\tau)] + k\cos(\Omega t). \tag{1}$$

For convenience, we can give the hyperbolic tangent function's Taylor series expansion at $x = 0$. Then we have:

$$\ddot{x} = -a\dot{x} - bx + c\left(x(t-\tau) - \frac{1}{3}x^3(t-\tau) + o(x^5(t-\tau))\right) + k\cos(\Omega t). \tag{2}$$

To the best of our knowledge, no result has been obtained for the stability of bifurcating periodic solutions of such system. Meanwhile, the Hassard's approach and the frequency domain approach are invalid to determine the bifurcating periodic solutions of the system (1). So the main goal of this paper is to study the above model and uses the average method [17,18] to derive the average quation so that we can solve the problem by analyzing its characteristic equation.

The rest of this paper is organized as follows. In Section 2, the analysis of the linear stability and the existence of Hopf bifurcation are given for the autonomous system of Eq. (2). In Section 3, we apply the perturbation techniques to find closed form solutions for periodic motion. To justify the theoretical analysis, some numerical examples are given in Section 4. Discussions and conclusions are given in Section 5.

2. Local stability and the existence of Hopf bifurcation in the autonomous system

In this section, we first discuss the local stability of system (1). We can easily see that, if an external stimuli is added to the system, then the stable regions of trivial solution of the autonomous system may become those of periodic solutions of the corresponding non-autonomous system. Hence, it is useful to study the system (1) in the absence of external stimuli. The linear equation of system (1) for $k = 0$ can be expressed as:

$$\ddot{x} = -a\dot{x} - bx + c\tanh[x(t-\tau)]. \tag{3}$$

Similar to that in [11], the locations of the fixed points are independent of the inertial terms. This means that the fixed points are the same as those found in previous works [17]. Hence all of the fixed points occur at $x = 0$. Then we can give the hyperbolic tangent function's Taylor series expansion at $x = 0$ as follows:

$$\ddot{x} = -a\dot{x} - bx + c\left(x(t-\tau) - \frac{1}{3}x^3(t-\tau) + o(x^5(t-\tau))\right). \tag{4}$$

In order to be able to study the effect of time delay on the dynamic behavior of the inertial single neuron system, it is more convenient to work with the equations in an abstract form. This can be done by introducing $X = [x_1, x_2]^{\mathrm{T}}$ and $X(t-\tau) = [x_1(t-\tau), x_2(t-\tau)]^{\mathrm{T}}$ into Eq. (4), where the superscript T denotes the transpose. Then Eq. (4) can be rewritten as follows:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -ax_2 - bx_1 + c(x_1(t-\tau) - \frac{1}{3}x_1^3(t-\tau) + o(x_1^5(t-\tau))). \end{cases} \tag{5}$$

Moreover, Eq. (5) can be transformed into the following delay-differential equation in standard notation

$$\dot{X}(t) = L_0X(t) + R_0X(t-\tau) + R_\alpha X^3(t-\tau) + F(X(t-\tau)), \tag{6}$$

where

$$L_0 = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix}, \qquad R_0 = \begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix},$$

$$R_\alpha = \begin{bmatrix} 0 & 0 \\ -d & 0 \end{bmatrix}, \qquad F(X(t-\tau)) = \begin{bmatrix} 0 & 0 \\ o(x_1^5(t-\tau)) & 0 \end{bmatrix},$$

$$d = \frac{1}{3}c.$$

Neglecting the above higher nonlinear terms, we obtain the following equation:

$$\dot{X}(t) = L_0X(t) + R_0X(t-\tau). \tag{7}$$

The stability of zero solution of Eq. (7) is determined by the eigenvalues of the Jacobian matrix of the linear part. The characteristic function of Eq. (7) can be obtained by substituting the trial solution $X(t) = C\exp(\lambda t)$ into its linear part, then the characteristic equations for system (7) are:

$$\begin{aligned} F_1(\lambda) &= \det(\lambda I - L_0 - R_0\mathrm{e}^{-\lambda\tau}) \\ &= \lambda(\lambda + a) + (b - c\mathrm{e}^{-\lambda\tau}) \\ &= \lambda^2 + a\lambda - c\mathrm{e}^{-\lambda\tau} + b = 0 \end{aligned} \tag{8}$$