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Nonlinear semigroup approach to age structured proliferating cell population with inherited cycle length

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Abstract

This paper deals with a nonlinear semigroup approach to semilinear initial-boundary value problems which model nonlinear age structured proliferating cell population dynamics. The model involves age-dependence and cell cycle length, and boundary conditions may contain compositions of nonlinear functions and trace of solutions. Hence the associated operators are not necessarily formulated in the form of continuous perturbations of linear operators. A family of equivalent norms is introduced to discuss local quasidissipativity of the operators and a generation theory for nonlinear semigroups is employed to construct solution operators. The resultant solution operators are obtained as nonlinear semigroups which are not quasicontractive but locally equi-Lipschitz continuous.

MSC: 47H20; 35Q80; 92D25

Keywords: Proliferating cell population; Cycle length; Nonlinear semigroup

1. Introduction

Of concern in this paper are semilinear initial-boundary value problems which model nonlinear age structured proliferating cell population dynamics of the form

$$\begin{aligned} &u_t(t,a,l) + u_a(t,a,l) + d(a,l,|u(t)|_{L^1})u(t,a,l) = 0, \quad t>0,\ (a,l)\in\Omega, \\ &u(0,a,l) = u_0(a,l), \quad (a,l)\in\Omega, \end{aligned} \tag{DE}$$

$$u(t,0,l) = 2\int_{\ell_1}^{\ell_2} k(l,\xi,|u(t)|_{L^1},u(t,\xi,\xi))\,\mathrm{d}\xi + 2c(l,|u(t)|_{L^1},u(t,l,l)), \quad (t,l)\in[0,\infty)\times(\ell_1,\ell_2). \tag{BC}$$

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Here $\Omega = \{(a, l) \in \mathbb{R}^2 : 0 < a < l,\ \ell_1 < l < \ell_2\}$, $|v|_{L^1}$ denotes the usual L^1 norm of $L^1(\Omega)$. $u(t, a, l)$ represents the density at time t of the population with age a and cell cycle length l. By cell cycle length we mean the time between cell birth and cell division. The function $d(\cdot, \cdot, \cdot)$ is the rate of cell mortality. The left-hand side of the boundary condition (BC) represents the population of the daughter cells with cell cycle length l which are born from mother cells with various cycle lengths.

Lebowitz and Rubinow [9] proposed the linear equation for modelling microbial population in terms of age and cycle length formalism. In their model the cell cycle length l of individual cell is assumed to be an inherent characteristic determined at birth. Webb [13] treated this linear equation in the space of continuous functions and showed that the solution has the asynchronous exponential growth. Latrach and Mokhtar-Kharroubi [7] have dealt with the spectral analysis of a linear operator corresponding to the linear problem in $L^p(\Omega)$. They show that the linear operator generates a C_0 semigroup in $L^p(\Omega)$ with compactness assumption on K. Boulanouar [2,3] demonstrated that the linear operator mentioned above generates a C_0 semigroup in $L^p(\Omega)$ without restriction and studied the irreducibility and the asymptotic behavior of the semigroup. Jeribi [5] and Latrach et al. [8] studied a stationary problem related to a nonlinear version of Lebowitz–Rubinow model.

Our objective here is to present a nonlinear version of Lebowitz–Rubinow model (DE)–(BC) as abstract semilinear problems

$$u'(t) = Au(t) + Fu(t), \quad t > 0, \quad u(0) = v \tag{SE}$$

in $X \equiv L^p(\Omega)$, $p \geqslant 1$, which are coupled with nonlinear constraints of the form

$$L_1 u(t) = K(L_2 u(t), u(t)), \quad t > 0 \tag{NC}$$

and to construct a semigroup, denoted $\{S(t) : t \geqslant 0\}$, of Lipschitz operators in $L^p(\Omega)$ which provides solutions in a strong or generalized sense to the evolution problem (SE)–(NC).

These constraints (NC) are formulated in a possibly different Banach space $Z \equiv L^p(\ell_1, \ell_2)$. Here A is a linear operator in X which represents a linear differential operator subject to suitable boundary conditions, L_1 and L_2 are linear operators from the domain $D(A)$ of A into Z, F is a possibly nonlinear but continuous perturbing operator in X and K is a nonlinear continuous operator from $Z \times X$ into Z which specifies the nonlinear constraints in Z. The operators L_1 and L_2 stand for unbounded operators such as trace operators in our concrete problem, and so they need not be continuous.

Although F and K are continuous and (SE) is a typical semilinear evolution equation in X, the problem for (SE)–(NC) cannot be formulated as a standard semilinear evolution problem, since (NC) contains a composition of K and an unbounded operator L_2. In fact, we necessitate treating the problem for (SE)–(NC) as a fully nonlinear evolution problem rather than a semilinear evolution problem. Moreover, the resulting semigroup $\{S(t) : t \geqslant 0\}$ is not quasicontractive with respect to the original norm $|\cdot|_X$ but it is locally equi-Lipschitz continuous on X in the sense that for any $\tau > 0$ and any bounded set B there exist constants $M \equiv M(\tau, B)$ and $\omega \equiv \omega(\tau, B)$ such that

$$|S(t)v - S(t)w|_X \leqslant M e^{\omega t} |v - w|_X$$

for $v, w \in B$ and $t \in [0, \tau]$.

The feature of our argument is to apply a generation theorem of quasicontractive nonlinear semigroup with respect to a family of equivalent norms $\{|\cdot|_\delta : \delta \geqslant 0\}$. In the case of $p > 1$, the linear operator A is restricted to set of elements v in $D(A)$ for which (NC) holds and the original problem is converted to a fully nonlinear problem which we call (NP; p) in this paper. In the case of $p = 1$, the problem is reformulated as a formal semilinear evolution problem in the product space $Z \times X$, although the semilinear operator is no longer a continuous perturbation of the linear unbounded operator. In both cases, appropriate equivalent norms should be employed to show the local quasidissipativity and subtangential conditions of the governing operators and then apply a generation theory for locally quasicontractive semigroups. For another approach to this type of semilinear problem we refer to [12].

Our paper is organized as follows: Section 2 outlines the main points of a generation theory for locally quasicontractive semigroups with respect to a family of equivalent norms. In Section 3, the fundamental assumptions for the models are made and a full statement of our main theorem (Theorem 3.1) is given. Basic lemmas which we need to prove our