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Exact controllability of nonlinear diffusion equations arising in reactor dynamics

K. Sakthivel[a], K. Balachandran[a], S.S. Sritharan[b,*]

[a]*Department of Mathematics, Bharathiar University, Coimbatore 641 046, India*
[b]*Department of Mathematics, University of Wyoming, Laramie WY 82071, USA*

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Abstract

This paper studies the problems of local exact controllability of nonlinear and global exact null controllability of linear parabolic integro-differential equations, respectively, with mixed and Neumann boundary data with distributed controls acting on a subdomain ω of $\Omega \subset R^n$. The proof of the linear problem relies on a Carleman-type estimate and observability inequality for the adjoint equations and that the nonlinear one, on the fixed point technique.

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1. Introduction

In the problem of partial integro-differential equation modeled by including the integral term with a well-known basic partial differential equation in various fields of physics and engineering, it is essential to take into account the effect of past history while describing the system as a function at a given time. Consider for example, a physical situation which gives rise to a parabolic partial integro-differential equation of the form

$$\begin{cases} \dfrac{\partial y}{\partial t} - \Delta y + \displaystyle\int_0^t a(t,r)g(y(r,x))\,\mathrm{d}r = f & (t,x) \in (0,T) \times \Omega, \\ y(0,x) = y_0(x), & x \in \Omega, \\ y(t,x) = 0 & (t,x) \in (0,T) \times \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega \subset R^n$ is a connected bounded domain with smooth boundary $\partial\Omega$, is a feedback heat control in the interior of some heat conducting medium, where the control mechanism possesses some inertia or a similar control situation for a reaction–diffusion problem. In the analysis of space time dependent nuclear reactor dynamics, if the effect of a linear temperature feedback is taken into consideration and the reactor model is considered as an infinite rod, then the

* Corresponding author. Tel.: +1 307 766 4221.
E-mail addresses: Sakthivel1980@gmail.com (K. Sakthivel), balachandran_k@lycos.com (K. Balachandran), Sri@uwyo.edu (S.S. Sritharan).

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one group neutron flux $y(t,x)$ and the temperature $v(t,x)$ in the reactor are given by the following coupled equations (see for instance [15,22] and also [18]):

$$\begin{cases} \dfrac{\partial y}{\partial t} - (a(x)y_x)_x = (c_1 v + c_2 - 1)\Sigma_f y(x) \quad (t > 0, -\infty < x < \infty), \\ \rho c \dfrac{\partial v}{\partial t} = c_3 \Sigma_g y, \end{cases} \tag{1.2}$$

where a is the diffusion coefficient and $\Sigma_f, \Sigma_g, \rho, c, c_i = (i = 1, 2, 3)$ are physical quantities. By integrating the second equation in (1.2) in the interval $(0, t)$ and substituting it into the first equation, we obtain the following nonlinear integro-differential diffusion equation:

$$\frac{\partial y}{\partial t} - (a(x)y_x)_x = \beta y \int_0^t y(r,x)\,\mathrm{d}r + by \quad (t > 0, -\infty < x < \infty), \tag{1.3}$$

where β, b are the constants associated with the initial temperature and various physical parameters. However, in the actual reactor systems, the temperature is a function of position x, which may be one, two or three dimensional. Thus it is more realistic to consider the heat equation for y in a higher dimensional spatial domain (see, for example, [2,20–22]). In this paper we consider a more general system of integro-differential equations of the form

$$\begin{cases} \dfrac{\partial y}{\partial t} - Ly = g(t,x) + f\left(t, x, y, \displaystyle\int_0^t K(t,x,r,y(r,x))\,\mathrm{d}r\right) & (t,x) \in (0,T) \times \Omega, \\ y(0,x) = y_0(x), & x \in \Omega, \\ d(x,t)\dfrac{\partial y}{\partial v} + y = 0 & (t,x) \in (0,T) \times \partial\Omega, \end{cases} \tag{1.4}$$

where Ω is a connected bounded domain in R^n with smooth boundary $\partial\Omega$ and

$$Ly = \sum_{i,j=1}^{n} a_{ij}(x) \frac{\partial^2 y}{\partial x_i \partial x_j} + \sum_{i=1}^{n} a_i(x) \frac{\partial y}{\partial x_i} - a_0(x) y.$$

The existence, uniqueness and asymptotic behavior of solutions of the system of the form (1.4) has been studied in [20]. This problem governs many physical systems occurring in diffusion problems and includes (1.2) as a special case. With reference to Pachpatte [20], other problems in nuclear reactor dynamics give rise to equations of the form (1.1) but with the integral term replaced by

$$\int_0^t k(t,r,x) y^2(r,x)\,\mathrm{d}r.$$

A general reference to solvability theory of the type of nonlinear integro-differential equations considered here is Vrabie [28]. Moreover one can refer the introduction of Yanik and Fairweather [29] for a rich set of integro-differential models and good number of references.

Controllability of dynamical systems represented by partial differential equations has been studied by several authors by using different techniques (see, [3,6] and the references cited therein). One such technique is duality arguments. By using this, the exact controllability of a linear system can be reduced to the observability estimate of its dual system. In the same way, the exact controllability of a semilinear system can be reduced to an estimate, provided the observability constant depends on the coefficients of the "linearized" systems. Thus, one of the main problems in the theory of exact controllability is how to construct the observability estimates for the linear system. Broadly, for deriving the observability estimates, we have the following three important methods: multiplier techniques [16,19], Carleman estimates [9,10,25] and microlocal analysis [7].

However, of all these methods so far the most effective method in establishing such a result is the method of Carleman-type estimates. There have been great concerns in the Carleman inequalities after the publication of the basic paper by Carleman in 1939. In particular, after the appearance of the fundamental results by Hörmander [11], this theory is one of the most developing areas of linear partial differential equations and also for the detailed results of Carleman inequalities for parabolic equations one can refer [12]. Using this technique the exact controllability of the heat equation