Available online at www.sciencedirect.com
ScienceDirect
Nonlinear Analysis: Real World Applications 9 (2008) 2086 – 2105
Nonlinear Analysis
Real World Applications
www.elsevier.com/locate/na

A reaction–diffusion system modeling predator–prey with prey-taxis

Bedr'Eddine Ainseba[a], Mostafa Bendahmane[b,*], Ahmed Noussair[c]

[a] Univesité Victor Segalen Bordeaux 2 IMB, UMR CNRS 5251 and INRIA Futurs Bordeaux, France
[b] Departamento de Ingenieria Matematica Universidad de Concepcion, Casilla 160-C, Concepcion, Chile
[c] Universite Bordeaux 1, IMB, 351 cours de la Libération, 33400 Talence, France

Received 2 May 2007; accepted 29 June 2007

Abstract

We are concerned with a system of nonlinear partial differential equations modeling the Lotka–Volterra interactions of predators and preys in the presence of prey-taxis and spatial diffusion. The spatial and temporal variations of the predator's velocity are determined by the prey gradient. We prove the existence of weak solutions by using Schauder fixed-point theorem and uniqueness via duality technique. The linearized stability around equilibrium is also studied. A finite volume scheme is build and numerical simulation show interesting phenomena of pattern formation.

MSC: 35K57; 35K55; 92B05

Keywords: Reaction–diffusion system; Predator–prey; Prey-taxis; Finite volume scheme

1. Introduction

This work is concerned with the mathematical and numerical analysis of a system of partial differential equations of reaction–diffusion–advection system. This system describes the local interactions of predators and preys with prey-taxis. Prey-taxis is a direct movement of predators in response to a variation of prey. In this paper we assume that, locally (i.e., at each point and each instant), predators attack preys following the familiar Lotka–Volterra interaction. Spatial dispersal of the prey is pure diffusion and the spatial-temporal variations of the predator's velocity are determined by the prey gradient. Several field studies measuring characteristics of individual movement confirm the basic hypothesis about the dependence of acceleration on a stimulus gradient (see [13] for instance). Understanding spatial and temporal behaviors of interacting species in ecological system is a central problem in population ecology. Various types of mathematical models have been proposed to study problems of coexistence or exclusion of competing species. The appearance of advection-driven heterogeneity in relation to single and multispecies ecological interactions was studied by Levin [7], Levin and Segel [8], Okubo [12], Mimura and Murray [10], Mimura and Kawasaki [9], Mimura and Yamaguti [11], and many other authors. In passing, we mention that in [2] (see also [4]) the authors have considered the

* Corresponding author. Tel.: +56 41 2203129; fax: +56 41 2522055.
E-mail addresses: bea@sm.u-bordeaux2.fr (B.E. Ainseba), mostafab@ing-mat.udec.cl (M. Bendahmane), noussair@math.u-bordeaux. fr (A. Noussair).

1468-1218/$ - see front matter
doi:10.1016/j.nonrwa.2007.06.017

interaction of two species assuming that both species attract the other by some devise. These studies form a theoretical basis for modeling complex spatio-temporal dynamics observed in real systems. Moreover, from mathematical point of view these models have a structure and remains very challenging.

Let first $u=u(t,x)$ and $v(t,x)$ represent the predator and prey population densities respectively at time t and position x. Let $r>0$ be the natural growth rate of prey, K be the carrying capacity, and let $-a$ $(a>0)$ be the natural exponential decay of the predator population. Then, we assume the logistical growth rate of prey reads $k(v)=rv(1-(v/K))$ and the predation rate reads $\pi(v)=pv/(1+qv)$ with $1/p$ the time spent by a predator to catch a prey and q/p the manipulation time, offering a saturation effect for large densities of preys when $q>0$. Last, e being the conversion rate from prey to predator.

Our model that governs the dynamics of a predator and prey system with prey-taxis is the following reaction–diffusion–advection system

$$\begin{cases} \partial_t u - d_1\Delta u + \operatorname{div}(u\chi(u)\nabla v) = -au + e\pi(v)u \quad \text{in } Q_T, \\ \partial_t v - d_2\Delta v = k(v) - \pi(v)u, \end{cases} \tag{1.1}$$

where $Q_T := \Omega \times (0,T)$, $T>0$ is a fixed time, and Ω is a bounded domain in $\mathbb{R}^N$, with smooth boundary $\partial\Omega$ and outer unit normal η.

We augment (1.1) with no-flux boundary conditions on $\Sigma_T := \partial\Omega \times (0,T)$,

$$\frac{\partial u}{\partial \eta} = 0, \quad \frac{\partial v}{\partial \eta} = 0, \tag{1.2}$$

and initial distributions in Ω

$$u(x,0) = u_0(x), \quad v(x,0) = v_0(x). \tag{1.3}$$

In the model above, $d_1>0$ and $d_2>0$ are their diffusion rates.

The predators are attracted by the preys and χ denotes their prey-tactic sensitivity. In this work, we assume at first that there exists a maximal density of predators, the threshold u_m, such that $\chi(u_m)=0$. Intuitively, this amounts to a switch to repulsion at high densities, sometimes referred to as volume-filling effect or prevention of overcrowding (see [5]). We refer also to [3] for some work in that direction for degenerate diffusion. This threshold condition has a clear biological interpretation: the predators stop to accumulate at a given point of Ω after their density attains certain threshold values and the prey-tactic cross diffusion $h(u)=u\chi(u)$ vanishes identically when $u \geqslant u_m$.

In this work we assume that the function χ in (1.1) satisfies

$$\chi \in C^2([0,1]) \quad \text{and} \quad \chi(u_m) = 0. \tag{1.4}$$

Before stating our main results, we give the definition of a weak solution.

Definition 1.1. A weak solution of (1.1)–(1.3) is a pair (u,v) of functions satisfying the following conditions, $u(t,x)\geqslant 0$ and $v(t,x)\geqslant 0$, for a.e. $(t,x)\in Q_T$,

$$\begin{aligned} &u \in L^\infty(Q_T) \cap L^2(0,T;H^1(\Omega)) \cap C(0,T,L^2(\Omega)), \\ &\partial_t u \in L^2(0,T;(H^1(\Omega))'), \quad u(0)=u_0, \\ &v \in L^\infty(Q_T) \cap L^p(0,T;W^{2,p}(\Omega)) \cap C(0,T,L^2(\Omega)) \quad \text{for all } p>1, \\ &\partial_t v \in L^2(Q_T), \quad v(0)=v_0, \end{aligned} \tag{1.5}$$

and, for all $\varphi, \psi \in L^2(0,T;H^1(\Omega))$,

$$\begin{cases} \int_0^T \langle \partial_t u, \varphi\rangle \,\mathrm{d}t + \iint_{Q_T} d_1 \nabla u \cdot \nabla\varphi - u\chi(u)\nabla v \cdot \nabla\varphi \,\mathrm{d}x\,\mathrm{d}t \\ \quad = \iint_{Q_T} (-a + e\pi(v))u\varphi \,\mathrm{d}x\,\mathrm{d}t, \\ \int_{Q_T} \partial_t v\psi + d_2\nabla v \cdot \nabla\psi \,\mathrm{d}x\,\mathrm{d}t = \iint_{Q_T} (k(v) - \pi(v)u)\psi \,\mathrm{d}x\,\mathrm{d}t, \end{cases} \tag{1.6}$$

where $\langle\cdot,\cdot\rangle$ denotes the duality pairing between $H^1(\Omega)$ and $(H^1(\Omega))'$.