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Permanence of a discrete multispecies Lotka–Volterra competition predator–prey system with delays

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Abstract

In this paper, we propose a discrete multispecies Lotka–Volterra competition predator–prey system with delays. For general nonautonomous case, sufficient conditions are established for the permanence of the system.

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Keywords: Permanence; Delay; Lotka–Volterra competition predator–prey system; Discrete; Nonautonomous

1. Introduction

In theoretical ecology, it is important whether or not all species in a multispecies community can be permanent. There have been many studies for the permanence of models governed by differential systems (see [2,7,9,11,13,18,21] and the references cited therein). For example, for two-species Lotka–Volterra differential systems, it is known that time delays are harmless for the permanence of a predator–prey system (see [21]) and a competition system (see [13,18]). On the other hand, in the last two decades, several papers (see, for example, [4,5,8,10,14,15]) have appeared on the permanence of discrete models of Lotka–Volterra type without delay. Recently, some scholars are starting studying the permanence of discrete Lotka–Volterra systems with delays (see [12,17,16,24] and the references cited therein).

Saito et al. [17] considered the permanence of the following discrete Lotka–Volterra competition system with delays:

$$x(n+1) = x(n) \cdot \exp\{r_1[1 - x(n-k_1) - \mu_1 y(n-k_2)]\},$$

$$y(n+1) = y(n) \cdot \exp\{r_2[1 - \mu_2 x(n-l_1) - y(n-l_2)]\}. \tag{1.1}$$

The initial condition of (1.1) is given as

$$x(-m) \geqslant 0, \quad m = 0, 1, \ldots, \max\{k_1, l_1\}, \quad x(0) > 0,$$

$$y(-m) \geqslant 0, \quad m = 0, 1, \ldots, \max\{k_2, l_2\}, \quad y(0) > 0. \tag{1.2}$$

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Here r_1, r_2, μ_1 and μ_2 are constants with $r_1 > 0, r_2 > 0, \mu_1 \geqslant 0$ and $\mu_2 \geqslant 0$, and delays k_1, k_2, l_1 and l_2 are nonnegative integers.

However, the biological population does not all rely on the mutual competition between populations to produce. There is massive population relationship between predators and preys. Now more and more scholars are realizing the importance of such relationship in biosphere. Recently, Saito et al. [16] further studied the permanence of the following discrete Lotka–Volterra predator–prey system with delays:

$$x(n+1) = x(n)\exp\left[r_1 - \sum_{j=1}^{m} a_{1j}x(n-k_{1j}) - \sum_{j=1}^{m} b_{1j}y(n-k_{2j})\right],$$

$$y(n+1) = y(n)\exp\left[r_2 + \sum_{j=1}^{m} a_{2j}x(n-l_{1j}) - \sum_{j=1}^{m} b_{2j}y(n-l_{2j})\right]. \tag{1.3}$$

The initial condition of (1.3) is given as

$$x(-v) \geqslant 0, \quad v = 0, 1, \ldots, k, \quad x(0) > 0,$$

$$y(-v) \geqslant 0, \quad v = 0, 1, \ldots, k, \quad y(0) > 0. \tag{1.4}$$

Here r_1 and r_2 are constants, and a_{ij} and b_{ij} $(i = 1, 2;\ j = 1, 2, \ldots, m)$ are nonnegative constants. Not all of a_{1j} and not all of b_{2j} $(j = 1, 2, \ldots, m)$ are zero. Delays k_{ij} and $l_{ij} (i = 1, 2;\ j = 1, 2, \ldots, m)$ are nonnegative integers.

However, there are few papers studying the permanence of discrete multispecies Lotka–Volterra predator–prey systems. Recently, Chen [3] first proposed the following discrete $n + m$-species Lotka–Volterra competition predator–prey system

$$x_i(k+1) = x_i(k)\exp\left\{b_i(k) - \sum_{l=1}^{n} a_{il}(k)x_l(k) - \sum_{l=1}^{m} c_{il}(k)y_l(k)\right\},$$

$$y_j(k+1) = y_j(k)\exp\left\{-r_j(k) + \sum_{l=1}^{n} d_{jl}(k)x_l(k) - \sum_{l=1}^{m} e_{jl}(k)y_l(k)\right\}, \tag{1.5}$$

sufficient conditions on the permanence and global stability of system (1.5) are obtained.

However, the discrete system (1.5) ignores the effect of more than one past generation. And the recent studies of the dynamics of natural populations indicate that the density-dependent population regulation probably takes place over many generations [6,20,19]. Note that Wang et al. [22] studied the following Lotka–Volterra model with delays:

$$x_i(k+1) = x_i(k)\exp\left\{r_i(k) - \sum_{j=1}^{n}\sum_{l=0}^{m} a_{ij}^{l}(k)x_j(k-l)\right\}, \quad i = 1, 2, \ \ , n. \tag{1.6}$$

They obtained the sufficient conditions which guarantee the global stability of system (1.6).

As was pointed out by Berryman [1], the dynamic relationship between predators and their preys has long been and will continue to be one of the dominant themes in both ecology and mathematical ecology due to its universal existence and importance. For the above reasons, we will consider the permanence of the following system (1.7) with delays. The model studied by this paper is

$$x_i(k+1) = x_i(k)\cdot\exp\left\{b_i(k) - \sum_{l=1}^{n}\sum_{s_1=0}^{h_1} a_{il}^{s_1}(k)x_l(k-s_1) - \sum_{l=1}^{m}\sum_{s_2=0}^{h_2} c_{il}^{s_2}(k)y_l(k-s_2)\right\},$$

$$y_j(k+1) = y_j(k)\cdot\exp\left\{-r_j(k) + \sum_{l=1}^{n}\sum_{s_3=0}^{h_3} d_{jl}^{s_3}(k)x_l(k-s_3) - \sum_{l=1}^{m}\sum_{s_4=0}^{h_4} e_{jl}^{s_4}(k)y_l(k-s_4)\right\}, \tag{1.7}$$