Available online at www.sciencedirect.com
ScienceDirect
Nonlinear Analysis: Real World Applications 10 (2009) 1–15
Nonlinear Analysis
Real World Applications
www.elsevier.com/locate/nonrwa

Bifurcation sequences of vibroimpact systems near a 1:2 strong resonance point

G.W. Luo[a,*], Y.L. Zhang[a], J.H. Xie[b]

[a] *School of Mechatronic Engineering, Lanzhou Jiaotong University, Lanzhou 730070, PR China*
[b] *Department of Applied Mechanics and Engineering, Southwest Jiaotong University, Chengdu, 610031, PR China*

Received 7 February 2007; accepted 9 August 2007

Abstract

Two vibroimpact systems are considered, which can exhibit symmetrical double-impact periodic motions under suitable system parameter conditions. Dynamics of such systems are studied by use of maps derived from the equations of motion, between impacts, supplemented by transition conditions at the instants of impacts. Two-parameter bifurcations of fixed points in the vibroimpact systems, associated with 1:2 strong resonance, are analyzed. Interesting features like Neimark–Sacker bifurcation of period-1 double-impact symmetrical motion, tangent bifurcation of period-2 four-impact motion, period-doubling bifurcation of period-2 four-impact motion and Neimark–Sacker bifurcation of period-4 eight-impact motion, etc., are found to occur near 1:2 resonance point of a vibroimpact system. The quasi-periodic attractor, associated with the fixed point of period-1 double-impact symmetrical motion, is destroyed as a tangent bifurcation of fixed points of period-2 four-impact motion occurs. However, for the other vibroimpact system the quasi-periodic attractor is restored via the collision of stable and unstable fixed points of period-2 four-impact motion. The results mean that there exist possibly more complicated bifurcation sequences of period-two cycle near 1:2 resonance points of non-linear dynamical systems.

Keywords: Vibration; Impact; Resonance; Periodic motion; Bifurcation

1. Introduction

Impact oscillators arise whenever the components of a vibrating system collide with rigid obstacles or with each other. Such systems with impacts exist in a wide variety of engineering applications, particularly in mechanisms and machines with clearances or gaps. Examples of these types of machines and equipment include heat exchangers, steam generator tubes, fuel rods in nuclear power plants, impacting hammers, hopping robots, rolling railway wheelset, gear transmissions and so on. The dynamics of vibroimpact systems are of considerable importance in noise suppression, reliability analysis and optimum design of machines with clearances or rigid obstacles. The trajectories of such systems in phase space have discontinuities caused by the impacts. The presence of the non-linearity and discontinuity complicates the dynamic analysis of such systems considerably, but they can be described theoretically

* Corresponding author. Fax: +86 931 4938613.
E-mail addresses: luogw@mail.lzjtu.cn, luogw@hotmail.com (G.W. Luo).

1468-1218/$ - see front matter
doi:10.1016/j.nonrwa.2007.08.027

and numerically by discontinuities in good agreement with reality. The broad interest in analyzing and understanding the performance of such systems is reflected by a still increasing amount of investigations devoted to this area. Several methods of the theoretical analysis have been developed and different models of impacts have been assumed in the past several years. Stability and bifurcations of different types of impact oscillators were reported in Refs. [1–7]. A special feature of impacting systems is the instability caused by low-velocity collisions, so-called grazing effects. The first important work in this area was done by Nordmark [8], who studied analytically the occurrence of singularities in a piecewise linear system. This work has been further expanded by thorough investigations of two-dimensional maps, where some universal behavior has been found [9–16]. Souza and Caldas [17] applied a model-based algorithm for the calculation of the spectrum of the Lyapunov exponents of attractors of mechanical systems with impacts. Chattering impact phenomenon was found to exist in an oscillator with limiting stops by Nguyen et al. [18]. Peterka [19] studied chaotic motion of an intermittency type of the impact oscillator appearing near segments of saddle-node stability boundaries of subharmonic motions with two different impacts in motion period. Shaw [20] studied periodic-impact motions and sliding bifurcation of a periodically forced impact oscillator with large dissipation. Luo and Gegg [21] developed the force criteria for stick and non-stick motions in harmonically forced, friction-induced oscillators from the local theory of non-smooth dynamical systems on connectable domains. Wagg [22,23] considered the rising phenomena which occur in sticking solutions of impact oscillators. Quasi-periodic motions in multiple degree-of-freedom oscillators with impacts were analyzed by numerical and analytical studies [24–27]. Hu [28] presented how to control the chaos of dynamical systems with discontinuous vector field through the paradigm of a harmonically forced oscillator having a set-up elastic stop. The algorithms of position control of impact oscillator and synchronization of two impact oscillators are demonstrated by Lee and Yan [29]. Souza et al. [30] proposed a feedback control method to suppress chaotic behavior in oscillators with limited power supply. Experimental study of base excited symmetrically piecewise linear oscillator was performed by Wiercigroch and Sin [31]. Along with the basic research into vibroimpact dynamics, a wide range of impacting models have been applied to simulate and analyze engineering systems operating within bounded dynamic responses. For example, in wheel-rail impacts of railway coaches [32], Jeffcott rotor with bearing clearance [33], pile driver [34], impact-forming machine [35], impact tools with progressive motions [36], impact dampers [37,38] and gears [39], etc., impacting models have proved to be useful [40].

The purpose of the present study is to focus attention on 1:2 strong resonance bifurcations of period-one double-impact symmetrical motion of vibroimpact systems. Notice that periodic-impact orbits and corresponding disturbed maps of some impact oscillators can be derived analytically by assuming that such systems possess proportional damping [1]. Consequently, one can analyze some special and complicated bifurcations of such systems by means of these disturbed maps explicitly, e.g., strong resonance bifurcations and high-codimension bifurcations, etc. Two typical vibroimpact systems are considered in the paper, which can exhibit symmetrical double-impact periodic motions under suitable system parameter conditions. Period-1 double-impact symmetrical motions of two vibroimpact systems are analytically derived by the equations of motions, impact law and the set of periodicity and matching conditions. The disturbed maps of the special periodic-impact motions can be expressed analytically by the disturbed differences at the instant of impacts. As a result, system parameters satisfying 1:2 strong resonance condition are obtained by computing and analyzing the eigenvalues of Jacobian matrix of the disturbed map. Two-parameter bifurcations of fixed points in the vibroimpact systems, associated with 1:2 strong resonance, are analyzed by use of impact Poincaré maps. Neimark–Sacker bifurcation of period-1 double-impact symmetrical motion, tangent and period-doubling bifurcations of period-2 four-impact orbits, Neimark–Sacker bifurcations of period-4 eight-impact motions, etc., are found to exist near 1:2 resonance point of a vibroimpact system. The quasi-periodic attractor, associated with the fixed point of period-1 double-impact symmetrical motion, is destroyed as a tangent bifurcation of fixed points of period-2 four-impact motion occurs. However, for the other vibroimpact system the quasi-periodic attractor is restored via the collision of stable and unstable fixed points of period-2 four-impact motion. The results imply that there exist possibly more complicated bifurcation sequences of period-2 cycle near 1:2 resonance points of non-linear dynamical systems.

2. Mechanical model of an impact damper

The mechanical model for an impact damper is shown schematically in Fig. 1. Ideally, this model comprises two systems: the primary one consists of a rigid body with mass M_1, a linear spring with stiffness K, and a viscous damper with damping constant C, while the secondary system is a point mass M_2. The clearance between two masses