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Modeling the survival of a resource-dependent population: Effects of toxicants (pollutants) emitted from external sources as well as formed by its precursors

J.B. Shukla[a,1], Shalini Sharma[b], B. Dubey[c,*], Prawal Sinha[b]

[a] Centre for Modelling, Environment and Development, 18-Navsheel Dham, Bithoor Road, Kanpur-208017, India
[b] Department of Mathematics, Indian Institute of Technology, Kanpur-208 016, India
[c] Mathematics Group, Birla Institute of Technology and Science, Pilani-333 031, India

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Abstract

In this paper, a nonlinear mathematical model is proposed and analyzed to study the survival of a resource-dependent population. It is assumed that this population and its resource are affected simultaneously by a toxicant (pollutant) emitted into the environment from external sources as well as formed by precursors of this population. It is shown that as the cumulative rates of emission and formation of the toxicant into the environment increase, the densities of population and its resource settle down to lower equilibria than their initial carrying capacities, and their magnitudes decrease as rates of emission and formation of the toxicant increase. On comparing different cases, it is noted that when population is not affected directly by the toxicant but only its resource is affected, the possibility of its survival is greater than the case when both are affected simultaneously. But for large emission rate of toxicant, the affected resource may be driven to extinction under certain conditions and the population which wholly depends on it may not survive for long even if it is not affected directly by the toxicant.

Keywords: Population; Resource; Toxicant; Precursor

1. Introduction

Due to the rapid growth of industrialization, various kinds of toxicants (pollutants) are discharged into both aquatic and terrestrial environments affecting living beings as well as their resources [7,14–16,18,24,25]. These toxicants may be emitted into the environment from different sources (e.g. industries, vehicles, etc.) as well as formed by precursors produced by population such as human beings. The precursor of a population represents an intermediate substance produced by human actions which is converted into a toxicant in the environment affecting various resources including plants and vegetations [7,14,15,20,24,25]. For example, precursors of acid rains could be emissions of various oxides into the environment by various human activities affecting both human population and its resources [14].

* Corresponding author. Tel.: +91 1596245073; fax: +91 1596244183.
E-mail address: bdubey@bits-pilani.ac.in (B. Dubey).
[1] Present address: Distinguished Professor, LNMIIT, Jaipur-303 012, India.

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Several investigators have studied the effects of toxicants on biological species using mathematical models, [1–5,8–11,19,21]. In particular, Hallam and collaborators [1,8–11] have proposed and analyzed mathematical models to study the effects of toxicants on biological species when these are emitted into the environment from external sources. Huaping and Zhein [12] proposed a mathematical model to study the effect of a toxicant on population of two competing species. In these investigations the effects of a toxicant simultaneously on growth rate and carrying capacity of the species have not been considered. However, Freedman and Shukla [5] proposed models to study the effects of a toxicant on single-species and predator–prey systems by assuming that the intrinsic growth rate of species decreases as the uptake concentration of the toxicant increases while its carrying capacity decreases with the environmental concentration of the toxicant. Shukla et al. [20] have studied the effects of primary and secondary pollutants on a renewable resource using the same consideration. Shukla et al. [19] have also studied the survival of two competing species in a polluted environment using similar assumptions and showed that the usual competitive outcomes may be altered in the presence of a toxicant (see also [2–4,20–23]).

It may be noted here that in the above mentioned studies, toxicants are emitted from external sources into the environment with constant rates and the emission rates do not depend upon human population density. In real situations, however, toxicants (pollutants) are emitted into the environment by various human actions (e.g. industrial, household, vehicular discharges, etc.) either directly or formed by its precursors which are population-density-dependent [21]. Rescigno [17] has modeled and analyzed the effect of a toxicant, which is formed only by precursor of the population and affecting itself. He, however, did not either considered the emission of toxicant from external sources or the formation of uptake phase in the species population which is very important from the physiological point of view. It may also be pointed here that the intrinsic growth rate of species is not only affected by uptake concentration but it is also affected by environmental concentration of the toxicant. Further, there may exist situations where both the species and its resource are affected simultaneously by the toxicant emitted into the environment from various sources as well as formed by precursors of population. In view of these considerations, in this paper, a nonlinear mathematical model is proposed and analyzed for the survival of a resource-dependent biological population (such as human beings) when both the population and its resource are affected by a toxicant emitted into the environment from external sources as well as formed by its precursors. In modeling process, the following assumptions are made.

1. The densities of biological population as well as its resource are assumed to be governed by generalized logistic equations with respective intrinsic growth rates and carrying capacities.
2. The toxicant is emitted into the environment at a constant rate (cumulative rate due to emissions from different sources such as industrial chimney, vehicles, etc.) and its concentration is augmented due to its formation from precursors of population.
3. The growth rate of the cumulative density of precursors is proportional to the species population density and it decreases due to natural factors as well as due to its transformation into the environmental toxicant.
4. The concentration of the toxicant in the environment decreases due to its simultaneous assimilation, (absorption, deposition, uptake etc.) by population and its resource biomass, the magnitudes being proportional to the environmental concentration of the toxicant as well as corresponding densities of population and its resource. These assimilated amounts affect the densities of population and its resource biomass in the following two ways.
   (i) A fraction of the corresponding assimilated amount becomes part of the uptake phase and this uptaken toxicant interacts with the population and the resource biomass through biophysical processes leading to corresponding decrease in intrinsic growth rate per capita densities.
   (ii) The remaining fraction of this assimilated amount of toxicant decreases the respective growth rate density of population and its resource biomass directly.
5. The environmental concentration of the toxicant decreases the carrying capacities of population as well as its resource biomass.
6. The concentrations of the toxicant in the environment as well as in its uptake phase decrease due to natural factors by an amount which is proportional to respective concentration in the two phases. Also due to recycling there may exist a transformation of the toxicant from uptake phase to environmental phase.

In the following we propose a general model governing the above mentioned problem and analyze it by using stability theory of differential equations [6,13].