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Chaotic dynamics of a discrete prey–predator model with Holling type II

H.N. Agiza*, E.M. ELabbasy, H. EL-Metwally, A.A. Elsadany

Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

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Abstract

A discrete-time prey–predator model with Holling type II is investigated. For this model, the existence and stability of three fixed points are analyzed. The bifurcation diagrams, phase portraits and Lyapunov exponents are obtained for different parameters of the model. The fractal dimension of a strange attractor of the model was also calculated. Numerical simulations show that the discrete model exhibits rich dynamics compared with the continuous model, which means that the present model is a chaotic, and complex one.

Keywords: Prey–predator model; Holling type II functional response; Chaotic behavior; Layapunov exponents; Fractal dimension

1. Introduction

It is well-known that the Lotka–Volterra prey–predator model is one of the fundamental population models. A predator–prey interaction has been described firstly by two pioneers Lotka (1924) [1] and Volterra (1926) [2] in two independent works. After them, more realistic prey–predator models were introduced by Holling suggesting three kinds of functional responses for different species to model the phenomena of predation [3]. The research dealing with interspecific interactions has mainly focused on continuous prey–predator models of two variables, where the dynamics include only stable equilibrium or limit cycles. Nevertheless, some works by Danca et al. [4], Jing and Yang [5], Liu and Xiao [6] and Elabbasy et al. [7] showed that, for the discrete-time prey–predator models the dynamics can produce a much richer set of patterns than those observed in continuous-time models. Also Summers et al. have examined four typical discrete-time ecosystem models under the effects of periodic forcing [8]. They found that a system which has simplistic behavior in its unforced state can assume chaotic behavior when subjected to periodic forcing, dependent on the values chosen for the controlling parameters; such a phenomenon is well-known in the physical sciences in the theory of nonlinear oscillators see [8]. Danca et al. [4] demonstrated that, the chaotic dynamics in a simple discrete-time prey–predator model with Holling I take place [4]. In previous work, we modified Danca et al. model [4] and studied the complex dynamics [7], and found that the modified model is more realistic than

* Corresponding author. Tel.: +20 105160714; fax: +20 502246781.
E-mail addresses: agizah@mans.edu.eg (H.N. Agiza), emelabbasy@mans.edu.eg (E.M. ELabbasy), helmetwally@mans.edu.eg (H. EL-Metwally), aelsadany1@yahoo.com (A.A. Elsadany).

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that of Danca et al. [4]. However, the same conclusion was obtained using another technique, in which Euler method was used by Jing and Yang [5] and Liu and Xiao [6].

One of the real life applications of the prey–predator model is the Lynx and its prey the snowshoe Hare study documented by the Hudson Bay company for the time interval 1845–1935 [9]. The extension of discrete prey–predator model to cover the Holling type II had a little attention in the discrete case till now, due to its complexities. Therefore, the present work aims to shed more light on this subject through analyzing the dynamic complexities in a discrete-time prey–predator model with the Hollings type II functional response. That is, we shall focus our attention on analyzing how the Holling type II response [3] affects the dynamic complexities of prey–predator interactions.

This paper is organized as follows: in Section 2, the discrete prey–predator model with Holling type II is formulated, then the existence and stability of three fixed points are derived. In Section 3, some values of the parameters, such that the model undergoes the flip bifurcation and the Hopf bifurcation in the interior R^2_+, were derived and also discussed. The numerical simulation of the analytic results, such as the bifurcation diagrams, strange attractors, Lyapunov exponents and fractal dimension were presented in Section 4. Finally, Section 5 draws the conclusion.

2. Model

The classical prey–predator system always be in the following form

$$\begin{aligned} &x'(t) = xq(x) - \alpha y p(x) \\ &y'(t) = (p(x) - \beta) y \\ &x(0), y(0) > 0, \end{aligned} \tag{1}$$

where x, y represent the prey and predator density, respectively. $p(x)$ is the so-called predator functional response and $\alpha, \beta > 0$ are the conversion and predator's death rates, respectively. If $p(x) = \frac{mx}{1+\varepsilon x}$, $q(x) = ax(1-x)$, then Eq. (1) becomes the following well-known prey–predator model with the Holling type II functional response [10]:

$$\begin{cases} x'(t) = ax(1-x) - \alpha \dfrac{mxy}{1+\varepsilon x} \\ y'(t) = \left(\dfrac{mx}{1+\varepsilon x} - \beta\right) y, \end{cases} \tag{2}$$

where a, m and ε are the positive parameters that stand for prey intrinsic growth parameter, half saturation parameter, limitation of the growth velocity of the predator population with increase in the number of prey, respectively. The above model (Eq. (2)) has been studied by many authors [11–13] and it was shown that, the dynamics include only stable equilibrium or limit cycles.

Another possible way to understand the complex problem of competition between two interacting species is by using discrete models [4]. In the present work we study the dynamics of discrete prey–predator model with Holling type II which has the following two difference equations:

$$T : \begin{cases} x_{n+1} = ax_n(1-x_n) - \dfrac{bx_n y_n}{1+\varepsilon x_n} \\ y_{n+1} = \dfrac{dx_n y_n}{1+\varepsilon x_n}, \end{cases} \tag{3}$$

where a, b, c and d are the nonnegative parameters. The map given by Eq. (3) is a noninvertible map of the plane. The study of the dynamical properties of the above map allows us to have information about the long-run behavior of prey–predator populations. Starting from given initial condition (x_0, y_0), the iteration of (3) uniquely determines a trajectory of the states of population output in the following form

$$(x(n), y(n)) = T^n(x_0, y_0),$$

where $n = 0, 1, 2, \ldots$.

3. The fixed points and their stability

In this section, we first determine the existence of the fixed points of map (3), then investigate their stability by calculating the eigenvalues for the variational matrix of (3) at each fixed point. To determine the fixed points we have