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About the synchronization of MEMs

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Abstract

The stabilization of motions of MEMs is a difficult and important problem for the users because of the many nonlinearities of the system and the advantage for the structure to work close to resonance for a better electro-mechanical coupling. In this paper, we investigate the synchronized motion of a one degree of freedom model of MEMs taking into account the nonlinear effect of the electric force. By using the theory of synchronization with $\mu = \frac{\delta V}{V_e}$ as small parameter, we propose a procedure to ensure the synchronized motion close to resonance and to investigate its asymptotic stability. To our knowledge, this problem has never been investigated.

Keywords: MEMs; Synchronization; Small parameter; Nonlinear effects

1. Introduction

For many years, several modeling of MEMs have been proposed and specific procedures are now programmed with softwares like ANSYS. Nonlinear effects (geometry, electrostatic forces . . .) are taken into account and coupled electo-mechanical calculations are also proposed. As noted in [7], the situation is not so clear concerning dynamic calculations and several investigations have to be carried out, for example about damping or dynamic electro-mechanical coupling effects. Dynamics of nonlinear systems may have complicated behavior from regular to chaotic one with every kind of instability (divergence, flutter, Hopf bifurcations, limit cycles . . .) (see [4,6,8,13,9], etc). Moreover, the MEMs are used close to resonance because of a better electro-mechanical coupling. However, that may induce the collapse or pull-in and the control of motion close to resonance is in fact one of the final aims in using MEMs. We propose here to contribute to such an aim by applying the theory of synchronization to the dynamics of MEMs. In order to present the essence of the method, we only use a one degree of freedom equivalent system. The limitations of such a model are well-known but we think that the higher forgetting lies in a frequent assumption consisting in supposing that the electrostatic force is "vertical" and is not a real pression. However, nonlinear nonuniform pression may induce effects like those induced by a follower force. Such effects are (often and here by nature of a one d.o.f. system) neglected and we focus only on nonlinear effects. Remark nevertheless that without neglecting the previously mentioned effects, static instability (divergence) (as pull-in is often presented) is

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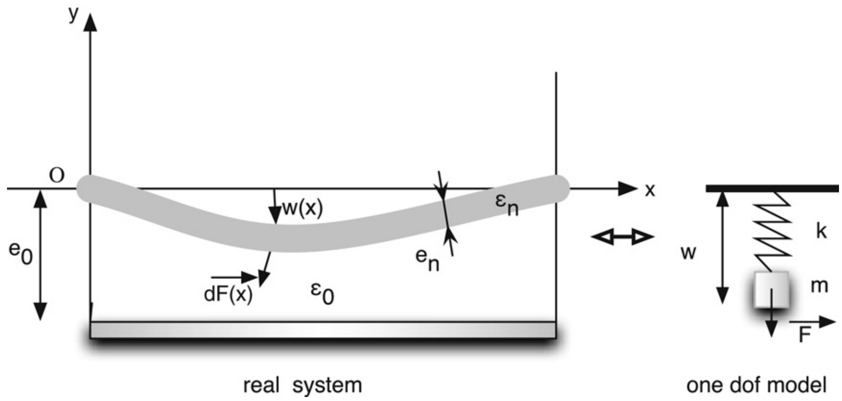

Fig. 1. The model.

not identical to the dynamic one (flutter) ([3,5,1], ...). That is also mentioned in [10] after simulations. The author mentions "that the system may become unstable before the static pull-in voltage due to the dynamical effect and thus one should consider the dynamic pull-in voltage".

In the first section the mechanical model is presented. The second section is devoted to the derivation of synchronization's equations. In the last part, MAPLE procedure allowing numerical simulations and experimental verifications are given.

2. The model and the equation

According to the mechanical model, the dynamic equations are:

$$m\ddot{v} + kv = F(v, V) \tag{1}$$

where m is the equivalent mass and k is the equivalent rigidity of the system. $F(v, V)$ denotes the electric force acting on the mass when a voltage V is applied and (see [2] for example):

$$F(v, V) = \frac{aV^2}{(1 + b(1 - v))^2} = V^2 H(v)$$

with a, b parameters depending on geometrical and physical data. This expression means that fringing effects are here neglected [12]. Because V is supposed not depending on v, coupling effects are neglected too. m and k are scalars containing as much as possible information about the real system and

$$H(v) = \frac{a}{(1 + b(1 - v))^2}. \tag{2}$$

For example, they are obtained after using of Rayleigh–Ritz Method and may take into account the boundary conditions. More precisely, we have got:

$$a = \frac{\epsilon_n^2 A}{2e_0\epsilon_0^2 e_n^2} = \frac{\epsilon_0 \tilde{\epsilon_n}^2 A}{2e_0 e_n^2} \tag{3}$$

$$b = \frac{\epsilon_n e_0}{\epsilon_0 e_n} = \frac{\tilde{\epsilon_n} e_0}{e_n} \tag{4}$$

where ϵ_0 is the dielectric constant of vacuum, ϵ_n is the dielectric constant of the matter, $\tilde{\epsilon_n}$ the relative dielectric constant of the matter, e_0 is the gap spacing of the cavity, e_n the thickness of the membrane and A the capacitor surface (see Fig. 1). $v = \frac{w}{e_0}$ is dimensionless and w is the displacement.

Let us suppose that an equilibrium position v_e is reached when the DC voltage V_e is applied. This means that:

$$kv_e = F(v_e, V_e) = V_e^2 H(v_e). \tag{5}$$