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Parameter identification problems and analysis of the impact of porous media in biofluid heat transfer in biological tissues during thermal therapy

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ABSTRACT

This paper considers time-dependent identification problems governed by some generalized transient bioheat transfer type models in biological systems with porous structures and directional blood flow. The analysis and the real applications of the porous media models and biofluid heat transfer in living tissues are relatively recent, however the blood perfusion rate and the porosity parameter affect considerably the effects of thermal physical properties on the transient temperature of biological tissues. The control considered in this work estimates simultaneously these two parameters. The result can be very beneficial for thermal diagnostics in medical practices, for example for laser surgery, photo and thermotherapy for regional hyperthermia, often used in the treatment of cancer. First, the mathematical models are introduced and the existence, the uniqueness and the regularity of the solution of the state equation are proved as well as stability and maximum principle under extra assumptions. Afterwards the identification problems with Tichonov regularization are formulated, in different situations, in order to control the online temperature given by radiometric measurement. An optimal solution is proven to exist and finally necessary optimality conditions are given. Some strategies for numerical realization based on the adjoint variables are provided.

1. Introduction

1.1. Statement of the problem

The mathematical problem studied in this paper derives from the modeling of the transport of thermal energy in biological systems with porous structures. The evaluation of thermal conductivities in living tissues is a very complex process which uses different phenomenological mechanisms including conduction, convection, radiation, metabolism, evaporation and others. Moreover blood flow and extracellular water affect considerably the heat transfer in the tissues and then the tissue thermal properties. The bioheat transfer process in tissues is dependent on the behavior of blood perfusion along the vascular system. The first model, taking account on the blood perfusion, was introduced by Pennes [1] in 1948 (see also Wissler [2] in 1998 where this paper of Pennes is revisited). The model is based on the classical thermal diffusion system, by incorporating the effects of metabolism and blood perfusion. The Pennes model has been adapted by many biologists for the analysis of various heat transfer phenomena in a living body. Others, after evaluations of the Pennes model in specific situations, have concluded that many of the hypotheses (foundational to the model) are not valid. Then these latter ones modified and generalized the model to adequate systems, see e.g. Chen and Holmes [3,4] in 1980; Chato [5] in 1980; Valvano et al. [6] in 1984; Weinbaum and Jiji [7] in 1985; Arkin et al. [8] in 1986; Hirst [9] in 1989 (see also e.g. Charney [10] in

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1992 for a review on mathematical modeling of the influence of blood perfusion). Recently, some studies have shown the important role of porous media in modeling flow and heat transfer in living body, and the pertinence of models including this parameter have been analyzed, see e.g. Shih et al. [11] in 2002; Khaled and Vafai [12] in 2003 and the references therein.

The goal of our contribution is to study time-dependent identification problems related to the effects of thermal and physical properties on the transient temperature of biological tissues with porous structures. More precisely we apply an optimal control technique to reconstruct (i.e. identify) the blood perfusion rate and the porosity parameter (these two parameters have a profound affect on the resulting temperature distribution).

To treat the system of motion in living body, we have written the transient bioheat transfer type model in a generalized form by taking into account the nature of the porous medium as follows (see an example of model below)

$$c(\varphi,x)\frac{\partial u}{\partial t} = div(\kappa(\varphi,x)\nabla u) - e(\varphi,x)p(u-u_a) - \mathrm{d}(\varphi,x)K(\vec{\vartheta},u) + r(\varphi,x)g + f, \quad \text{a.e. in } \mathcal{Q} = \Omega\times(0,T), \tag{1.1}$$

subject to the Dirichlet boundary condition
$u(x,t) = u_b(t) \quad in\ \Sigma = \partial\Omega\times(0,T)$,
and the initial condition
$u(x,0) = u_0(x) \quad in\ \Omega$,

under the pointwise constraints

$$\begin{aligned} &a_1 \le p \le a_2 \quad \text{a.e. in } \mathcal{Q},\\ &b_1 \le \varphi \le b_2, \end{aligned} \tag{1.2}$$

where the state function u is the temperature distribution, $T > 0$ is a fixed constant (a given final time), the body Ω is an open bounded domain in $\mathbb{R}^m$, $m \le 3$ with a smooth boundary $\Gamma = \partial\Omega$ which is sufficiently regular, and Ω is totally on one side of Γ, $\mathbf{n}$ is the unit outward normal to Γ and a_i, b_i, for $i = 1, 2$, are given positive constants. The quantity p is the blood perfusion rate and φ describes the porosity that is defined as the ratio of blood volume to the total volume (i.e. the sum of the tissue domain and the blood domain). The heat capacity type function $c(\varphi, .)$ and the thermal conductivity type function of the tissue $\kappa(\varphi, .)$ are assumed to be variable and satisfy $\nu \ge \kappa(\varphi, .) = \sigma^2(\varphi, .) \ge \mu > 0$, $M_1 \ge c(\varphi, .) = \lambda^2(\varphi, .) \ge M_0 > 0$ (where ν, μ, M_0, M_1 are positive constants). The second term on the right of the state Eq. (1.1) describes the heat transport between the tissue and microcirculatory blood perfusion, the third term K is corresponding, for example, to the directional convective mechanism of heat transfer due to blood flow, the last terms are corresponding to the sum of the body heating function which describes the physical properties of material (depending on the thermal absorptivity, on the current density and on the electric field intensity that can be calculated from the Maxwell equations) and the source terms that describe a distributed energy source which can be generated through a variety of sources, such as focused ultrasound, radio-frequency, microwave, resistive heating, laser beams and others (depending on the difference between the energy generated by the metabolic processes and the heat exchanged between, for example, the electrode and the tissue). The function u_a is the arterial blood temperature and is assumed to be in $L^\infty(\mathcal{Q})$, the function u_b is the boundary temperature and is assumed to be independent on space variable and sufficiently regular in time (for example in $C^1([0,T])$). The function u_0 is the initial value and is assumed to be variable. The vector function $\vec{\vartheta}$ is the flow velocity which satisfies the regularity: $\vec{\vartheta} \in L^\infty(0,T,W^{1,\infty}(\Omega))$.

1.2. Thermal damage calculations

After obtaining the temperature distribution, we can calculate the accumulation of thermal damage associated with the injury of tissue. For this, we can use the well known Arrhennuis damage integral formulation (see e.g. Tropea and Lee [13] in 1992):

$$D(x,\tau_{\exp}) = \ln\left(\frac{C(0)}{C(\tau_{\exp})}\right) = A\int_0^{\tau_{\exp}} \exp\left(\frac{-E}{Ru(x,t)}\right)\mathrm{d}t, \tag{1.3}$$

where D is the nondimensional degree of tissue injury, C is the concentration of living cells, $\tau_{\exp}$ is the duration of the exposure. A is the molecular collision frequency (s^{-1}), E is the denaturation activation energy (J mol^{-1}) and R is the universal Gaz constant equal to 8.314 J mol^{-1} K^{-1}. The parameters A and E are dependent on the type of tissue, and the cumulative damage can be interpreted as the fraction of hypothetical indicator molecules that are denatured.

1.3. Example of model

The blood-perfused tumor tissue volume, including blood flow in microvascular bed with the blood flow direction, contains many vessels and can be regarded as a porous medium consisting of a tumor tissue (a solid domain) fully filled with blood (a liquid domain), see Fig. 1. Consequently the temperature distribution in biological tissue can be modelized by analyzing a conjugate heat transfer problem with the porous medium theory. For the tumor tissue domain, we use the Pennes bioheat transfer equation by taking account on the blood perfusion in the energy balance for the blood phase. For