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Qualitative analysis of a modified Leslie–Gower and Holling-type II predator–prey model with state dependent impulsive effects[☆]

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ABSTRACT

In this paper, we present a two-dimensional autonomous dynamical system modeling a predator–prey food chain which is based on a modified version of the Leslie–Gower scheme and on the Holling-type II scheme with state dependent impulsive effects. By using the Poincaré map, some conditions for the existence and stability of semi-trivial solution and positive periodic solution are obtained. Numerical results are carried out to illustrate the feasibility of our main results.

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1. Introduction

In the last few decades, impulsive differential equations (IDEs) have been extensively used as models in biology, physics, chemistry, engineering and other sciences, with particular emphasis on population dynamics. Many evolution processes in nature are submitted to short temporary perturbations that are negligible compared to the process duration. These short-time perturbations are often assumed to be in the form of impulses in the modeling process. Consequently, IDEs provide a natural description of such processes. In recent years, some IDEs have been introduced in population dynamics (see [1–14] and references therein), such as vaccination, chemotherapeutic treatment of disease, chemostat, birth pulse, control and optimization, etc. The majority of them just concern the systems with impulses at fixed times. However, impulsive state feedback control strategy is used widely in real life problems. In practical ecological systems, the control measures (by catching, poisoning or releasing the natural enemy, etc.) are taken only when the amount of species reaches a threshold value, rather than the usual impulsive fixed-time control strategy. Recently, a few studies on IDEs with state-dependent impulsive effects were made in [15,16,4,17–20]. In particular, Jiang and Lu [15,16] obtained the sufficient conditions of existence and stability of semi-trivial solution, and positive periodic solution for some systems by using the Poincaré map.

On the other hand, a two-dimensional system of autonomous differential equations modeling a predator–prey system, which incorporates a modified version of the Leslie–Gower functional response, that is the Holling-type II. The system

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describes a prey population x which serves as food for a predator y, and it can be written as follows:

$$\begin{cases} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} = \left[r_1 - b_1 x(t) - \dfrac{a_1 y(t)}{x(t)+k_1}\right] x(t) \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = \left[r_2 - \dfrac{a_2 y(t)}{x(t)+k_2}\right] y(t), \end{cases} \tag{1.1}$$

where x and y represent the population densities at time t; b_1, r_i, a_i and k_i ($i = 1, 2$) are model parameters assuming only positive values. The dynamic behaviors for system (1.1) with impulsive effects at fixed times or not, which have been studied extensively in the literature. For example, Aziz-Alaoui [21] and Nindjin [22] studied system (1.1) and obtained the sufficient condition for boundedness of solutions, existence of an attracting set and global stability of the coexisting interior equilibrium, and Song [2] and Liu [10] considered system (1.1) with impulsive effects at fixed times and established the conditions for the linear stability of trivial periodic solution and semi-trivial periodic solutions, the permanence and existence of a stable pest-eradication periodic solution.

Motivated by the above works, in this paper, we consider the dynamic behaviors for system (1.1) with state dependent impulsive effects. The system is modeled by the following equations:

$$\begin{cases} \left.\begin{aligned} \dfrac{\mathrm{d}x(t)}{\mathrm{d}t} &= \left[r_1 - b_1 x(t) - \dfrac{a_1 y(t)}{x(t)+k_1}\right] x(t) \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} &= \left[r_2 - \dfrac{a_2 y(t)}{x(t)+k_2}\right] y(t) \end{aligned}\right\} \quad x \neq h, \\ \left.\begin{aligned} \Delta x(t) &= x(t^+) - x(t) = -p x(t) \\ \Delta y(t) &= y(t^+) - y(t) = q y(t) + \alpha \end{aligned}\right\} \quad x = h, \end{cases} \tag{1.2}$$

where $h \in (0, \infty)$, $p \in (0, 1)$ and $q \in (-1, \infty)$. When the amount of the prey reaches the threshold h at time t_h, controlling measures are taken and the amount of prey and predator abruptly turn to $(1-p)h$ and $(1+q)y(t_h) + \alpha$, respectively.

This paper is organized as follows. In the next section, as preliminaries, we present some basic definitions, two Poincaré maps and an important lemma. In Section 3, we state and prove a general criterion for the semi-trivial periodic solution and positive periodic solution. Some specific examples are given to illustrate our results in the last section.

2. Preliminaries

The dynamic behaviors for system (1.1) clearly have an unstable focus $(0, 0)$ and two saddle $(r_1/b_1, 0)$ and $(0, r_2k_2/a_2)$ and one locally stable focus (x^*, y^*) under the following condition

(H) $r_1 \le r_2, k_1 \ge k_2$ and $r_2k_2/a_2 < r_1k_1/a_1$,

where

$$x^* = \frac{1}{2a_2b_1}\left\{-(a_1r_2 - a_2r_1 + a_2b_1k_1) + [(a_1r_2 - a_2r_1 + a_2b_1k_1)^2 - 4a_2b_1(a_1r_2k_2 - a_2r_1k_1)]^{\frac{1}{2}}\right\}, \tag{2.1}$$
$$y^* = \frac{r_2(x^* + k_2)}{a_2}.$$

Throughout in this paper, we assume that (H) is held. By the biological background of system (1.2), we only consider system (1.2) in the biological meaning region $D = \{(x, y) : x \ge 0, y \ge 0\}$. Obviously, the global existence and uniqueness of solutions of system (1.2) are guaranteed by the smoothness properties of f, which denotes the mapping defined by right-side of system (1.2) — for details see Lakshmikantham et al. [23], Bainov and Simeonov [24].

Set $R = (-\infty, \infty)$. First, we give the notion of the distance between a point and a set. It is defined as follows. Let $S \in R^2 = \{(x, y) : x \in R, y \in R\}$ be an arbitrary set and $P \in R^2$ be an arbitrary point. Then the distance between the point P and the set S is denoted by

$$d(P, S) = \inf_{P_0 \in S} |P - P_0|.$$

Let $z(t) = (x(t), y(t))$ be any solution of (1.2). Next, we define the positive orbit through the point $z_0 \in R^2_+ = \{(x, y) : x \ge 0, y \ge 0\}$ for $t \ge t_0$ as:

$$O^+(z_0, t_0) = \{z \in R^2_+ : z = z(t), t \ge t_0, z(t_0) = z_0\}.$$

In order for the convenience of statement, in the rest of this paper, we introduce the definitions:

Definition 2.1 (*Orbital Stability*). $z^*(t)$ is said to be orbitally stable, if given $\varepsilon > 0$, there exists $\delta = \delta(\varepsilon) > 0$ such that, for any other solution, $z(t)$, of system (1.2) satisfying $|z^*(t_0) - z(t_0)| < \delta$, then $d(z(t), O^+(z_0, t_0)) < \varepsilon$ for $t > t_0$.

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