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Bifurcation analysis in a delayed diffusive Nicholson's blowflies equation[☆]

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ABSTRACT

The dynamics of a diffusive Nicholson's blowflies equation with a finite delay and Dirichlet boundary condition have been investigated in this paper. The occurrence of steady state bifurcation with the changes of parameter is proved by applying phase plane ideas. The existence of Hopf bifurcation at the positive equilibrium with the changes of specify parameters is obtained, and the phenomenon that the unstable positive equilibrium state without dispersion may become stable with dispersion under certain conditions is found by analyzing the distribution of the eigenvalues. By the theory of normal form and center manifold, an explicit algorithm for determining the direction of the Hopf bifurcation and stability of the bifurcating periodic solutions are derived.

1. Introduction

In order to describe the population dynamics of Nicholson's blowflies, Gurney et al. [1] have proposed the following delay equation

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -d_m u(t) + \varepsilon u(t-\tau)\mathrm{e}^{-au(t-\tau)}, \tag{1.1}$$

where ε is the maximum per capita daily egg production rate, $1/a$ is the size at which the blowfly population reproduces at its maximum rate, d_m is the per capita daily adult death rate and τ is the generation time. Eq. (1.1) has been extensively studied in the literature, where its results mainly concern the global attractivity of positive equilibrium and oscillatory behaviors of solutions (see [2–7,30,32]). Several studies have also been carried out on Eq. (1.1) with time periodic coefficients (see [8,9]) and on discrete Nicholson's blowflies equation (see [10–15]). After rescaling Eq. (1.1), it takes the form

$$\tilde{u} = au, \qquad \tilde{t} = \frac{t}{\tau}, \qquad \tilde{\tau} = d_m\tau, \qquad \beta = \frac{\varepsilon}{d_m},$$

and by dropping the tildes, then it may be written as

$$\frac{\mathrm{d}u}{\mathrm{d}t}(t) = -\tau u(t) + \beta\tau u(t-1)\mathrm{e}^{-u(t-1)}. \tag{1.2}$$

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To explain interactions among organisms, Yang and So [16] extended Eq. (1.2) to the flowing diffusive form

$$\frac{\partial u}{\partial t}(t,x) = d\triangle u(t,x) - \tau u(t,x) + \beta\tau u(t-1,x)e^{-u(t-1,x)}. \tag{1.3}$$

Furthermore, some researchers have studied the phenomenon by using an equation in the following form

$$\frac{du}{dt}(t,x) = D_m\Delta u(t,x) - d_m u(t,x) + \varepsilon u(t-\tau,x)e^{-au(t-\tau,x)}. \tag{1.4}$$

So and Yang [17] investigated global attractivity of the equilibrium of Eq. (1.3) with the Dirichlet boundary condition

$$u(t,0) = u(t,\pi) = 0, \quad \text{for } t \geq 0 \tag{1.5}$$

and they [16] studied the stability and existence of Hopf bifurcation of Eq. (1.3) with Neuman boundary condition. Some numerical and Hopf bifurcation analysis on Eq. (1.3) has been carried out by So, Wu and Yang [18]. Generalized Nicholson's blowflies models with distributed delay in Eqs. (1.3) and (1.4) have been studied extensively (see [19–22]). For the Dirichlet boundary value problem of the diffusive Nicholson's blowflies equation, So and Yang [17] have proved that there is a unique positive steady state solution if and only if $(\beta-1)\tau > \lambda_1$, where λ_1 is the principal eigenvalue of $-\triangle$ with Dirichlet boundary condition.

The purpose of the present paper is to study the bifurcation of Eq. (1.3) with Dirichlet boundary condition (1.5). We prove the existence of positive steady state bifurcation by a direct calculation presented in Robinson [23]. The conclusions are that the problem (1.3) and (1.5) has a unique positive steady state if and only if $\beta > 1 + d/\tau$, and the Eq. (1.3) with Dirichlet boundary condition

$$u(t,0) = u(t,\pi) = \ln\beta, \quad \text{for } t \geq 0 \tag{1.6}$$

has no other positive steady state solution except $u = \ln\beta$. On the other hand, we provide a detailed analysis of Hopf bifurcation for the problems (1.3) and (1.6) by applying the local Hopf bifurcation theory (see [24]). More specifically, we prove that, as β increases, the positive equilibrium $u^* = \ln\beta$ loses its stability and a sequence of Hopf bifurcations occur at u^*. Furthermore, by using the center manifold theory introduced by Lin, So and Wu [25] and normal form method due to Faria [26], we derive an explicit algorithm for determining the stability and direction of the Hopf bifurcations occurring at u^*.

The rest of this paper is organized as follows. In Section 2, existence of positive steady state bifurcation is established. In Section 3, the occurrence of Hopf bifurcation and the phenomenon that the unstable positive equilibrium state without dispersion may become stable with dispersion under certain conditions are found by analyzing the distribution of the eigenvalues. In Section 4, an algorithm for determining the direction and stability of the Hopf bifurcation is derived by using the center manifold due to Lin, So and Wu [25] and normal form method due to Faria [26]. Finally, some numerical analysis is given in order to illustrate the theoretic results found.

2. Positive steady state bifurcation

In the present section, we consider equation

$$\frac{\partial u}{\partial t}(t,x) = d\frac{\partial^2 u}{\partial x^2}(t,x) - \tau u(t,x) + \beta\tau u(t-1,x)e^{-u(t-1,x)}, \tag{2.1}$$

where $(t,x) \in D = (0,\infty)\times[0,\pi]$, $\beta,\ d,\ \tau > 0$, with Dirichlet boundary condition

$$u(t,0) = u(t,\pi) = 0, \quad \text{for } t \geq 0. \tag{2.2}$$

The steady state $u(x)$ of (2.1) and (2.2) satisfies

$$\begin{aligned} &du_{xx} = \tau u - \tau\beta u e^{-u},\\ &u(0) = u(\pi) = 0. \end{aligned} \tag{2.3}$$

Taking $v = u_x$, we can rewrite the equation in (2.3) into a pair of differential equations:

$$\begin{cases} u_x = v,\\ v_x = \dfrac{\tau}{d}(u - \beta u e^{-u}). \end{cases} \tag{2.4}$$

We can now apply phase plane ideas, treating x as the time variable. It follows from Eq. (2.4) that, on any trajectory,

$$\frac{d}{2}v^2 - \frac{\tau}{2}u^2 - \tau\beta(ue^{-u} + e^{-u}) = C,$$

where C is a constant. It is obvious that Eq. (2.4) has two fixed points given by

$$(u,v) = (0,0),\ (\ln\beta, 0).$$