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# Dynamics of a ratio-dependent eco-epidemiological system with prey harvesting

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In this article, we study a ratio-dependent eco-epidemiological system where prey population is subjected to harvesting. Mathematical results like positive invariance, boundedness, stability of equilibria, and permanence of the system have been established. The dynamics of zero equilibria have been thoroughly investigated to find out conditions on the system parameters such that trajectories starting from the domain of interest can reach a zero equilibrium following any fixed direction. We have also studied suitable conditions for non-existence of a periodic solution around the interior equilibrium. Computer simulations have been carried out to illustrate different analytical findings.



## 1. Introduction

The standard Lotka–Volterra type models, on which nearly all existing theory is built, assume that the per capita rate of predation depends on the prey numbers only. That means that the predators' functional response would be a function of prey density only. An alternative assumption is that, as the number of predators changes slowly (relative to prey change), there is often competition among the predators, and the per capita rate of predation depends on the numbers of both prey and predator, most likely and simply on their ratio. That is, in general, predator functional response should certainly be a function of prey and predator densities [1,2].

It is well known that the classical prey-dependent predator–prey model exhibits the well known "paradox of enrichment" [3,4], which states that enriching a predator–prey system (by increasing the carrying capacity) will cause an increase in the equilibrium density of the predator but not in that of the prey and will destabilize the positive equilibrium (the positive steady state changes from stable to unstable as the carrying capacity increases) and thus increases the possibility of stochastic extinction of the predator. However, numerous field observations provide contrary to this paradox of enrichment. It is often observed in nature that fertilization increases the prey density, but does not destabilize a stable steady state and fails to increase the amplitude of the oscillations in systems that already cycle [5]. Another paradox that the predator–prey model with Michaelis–Menten functional response exhibits is the so-called "biological control paradox" [6], which states that we cannot have both a low and stable prey equilibrium density. However, there are many examples of successful biological control where the prey is maintained at very low densities compared with its carrying capacity [7].

Recently, many biologists argue that when predators have to search, compete and share for food then the functional response in a predator–prey interaction should be ratio-dependent instead of classical prey-dependent. This has been

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strongly supported by numerous field and laboratory experiments and observations [1,7–9]. Ratio-dependent predator–prey model with Michaelis–Menten functional response has been studied by different researchers ([10–13] and others). Analysis shows that the ratio-dependent predator–prey model does not show the paradox of enrichment and the biological control paradox [13–15]. Kuang and Beretta [10] observed that ratio-dependent predator–prey models are richer in boundary dynamics and showed that if the positive steady state of the system is locally asymptotically stable, then the system has no non-trivial positive periodic solutions. Jost et al. [11] demonstrated that the equilibrium $(0, 0)$ for a ratio-dependent predator–prey model can either be a saddle point or an attractor and mutual extinction of prey and predator is also possible. Xiao and Ruan [12] and Berezovskaya et al. [14] observed that there exist different kinds of topological structures in the vicinity of the origin of a ratio-dependent predator–prey model. Hsu et al. [16] considered a ratio-dependent food chain model and studied the extinction dynamics as well as the sensitivity of the system to initial population. Berezovskaya et al. [15] presented an algorithmic approach to analyze the behavior of a ratio-dependent predator–prey system.

The effect of disease in ecological system is an important issue from mathematical and ecological points of views. Researchers are paying more and more interest to study the effect of disease on ecological system ([17–28] and the references therein). Arino et al. [29] considered the effect of disease in a ratio-dependent predator–prey model and investigated critically different topological structures around the origin. Successful invasion of a parasite into a host population and resulting host-parasite dynamics can depend crucially on other mechanisms of a host's community such as harvesting. Objectives of harvesting in a predator–prey system may be two fold. The primary objective is optimal exploitation of the harvested stock to maximize the profit ([30–36]). In contrast, Tu and William [37], Azar et al. [38], Dai and Tang [39], Bairagi et al. [40] considered harvesting to control predator population for the stability of the ecosystem. Xiao and Jennings [41] studied the dynamical properties of a ratio-dependent predator–prey model with constant rate of harvesting and showed different kinds of bifurcation. As far as knowledge goes, nobody has explicitly put a harvesting term in a ratio-dependent predator–prey-parasite model. The present paper is aimed at to formulate a ratio-dependent eco-epidemiological model where the prey population is subjected to harvesting, and study the dynamical behavior of the system around different equilibrium points including the zero one.

The organization of the paper is as follows: Section 2 deals with the model development. Mathematical results of the proposed model have been discussed in Section 3. Numerical studies have been given in Section 4 and finally, a brief discussion has been presented in Section 5.

## 2. Model development

### 2.1. Ratio-dependent predator–prey model

The classical model for a prey-dependent predator–prey system with a Michaelis–Menten functional response and logistic prey growth is governed by the equations

$$\begin{aligned} \frac{\mathrm{d}N}{\mathrm{d}t} &= rN\left(1-\frac{N}{k}\right)-\frac{mNP}{a+N} \\ \frac{\mathrm{d}P}{\mathrm{d}t} &= \frac{\alpha mNP}{a+N}-dp, \end{aligned} \tag{2.1.1}$$

where $N$ and $P$ are, respectively, the prey and predator densities, $r$ is the intrinsic growth rate of prey, $k$ is the carrying capacity of the environment, $m$ is the search rate, $a$ is the half-saturation constant, $\alpha$ is the conversion efficiency and $d$ is the death rate of predator population.

Based on the ratio-dependent theory, the system (2.1.1) can be written as

$$\begin{aligned} \frac{\mathrm{d}N}{\mathrm{d}t} &= rN\left(1-\frac{N}{k}\right)-\frac{mNP}{N+aP} \\ \frac{\mathrm{d}P}{\mathrm{d}t} &= \frac{\alpha mNP}{N+aP}-dp. \end{aligned} \tag{2.1.2}$$

### 2.2. Ratio-dependent eco-epidemiological model with prey harvesting

In the presence of infection, the prey population is divided into two classes, namely, susceptible population $(S)$ and infected population $(I)$, so that at any time $t$ the total prey population is given by $N(t) = S(t) + I(t)$.

- It is assumed that only susceptible population is capable of reproducing, but the infected population dies before having the capability of reproducing [42,43]. However, the infected population, $I$, still contributes with $S$ to population growth towards the carrying capacity.
- The mode of disease transmission is assumed to follow the simple law of mass action.
- The disease is spread among the prey population only and is not genetically inherited.
- It is also assumed that the infected population does not recover or become immune.