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Permanence and global stability for nonautonomous N-species Lotka–Valterra competitive system with impulses[☆]

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ABSTRACT

In this paper, we consider a class of nonautonomoua N-species Lotka–Volterra competitive systems with impulsive effects. Some new criteria on the permanence and global attractivity are established by developing the methods given in [Z. Teng and Z. Li, Permanence and asymptotic behavior of the N-species nonautonomous Lotka–Volterra competitive systems, Comput. Math. Appl. 39 (2000) 107–116; S. Ahmad and I.M. Stamovab, Asymptotic stability of an N-dimensional impulsive competitive system, Nonlinear Anal. RWA 8 (2007) 654–663].

1. Introduction

In the practical world, owing to many natural and man-made factors (e.g., fire, drought, flooding, crop-dusting, deforestation, hunting, harvesting etc.), the intrinsic discipline of biological species or ecological environment usually undergo some discrete changes of relatively short duration at some fixed times. Such sudden changes can often be characterized mathematically in the form of impulses. With the development of the theory of impulsive differential equations (the fundamental theory of impulsive differential equations can be seen in the monographs [1–4]), we can establish adequate mathematical models of impulsive differential equations to investigate the dynamic behaviors of such ecosystems with impulsive effects.

In recent years, various population dynamical models of impulsive differential equations have been proposed and studied extensively. Many important and interesting results on the dynamical behaviors for such systems, for example, the permanence, persistence, extinction, global stability, the existence of positive periodic solutions, bifurcation and dynamical complexity, etc., can be found in [5–17] and the references cited therein. In [14], the authors considered the following ω-periodic two-species Lotka–Volterra competitive system with impulsive effects

$$\begin{aligned} &\left.\begin{aligned}\dot{u}(t) &= u(t)(b_1(t) - a_{11}(t)u(t) - a_{12}(t)v(t)),\\ \dot{v}(t) &= v(t)(b_2(t) - a_{21}(t)u(t) - a_{22}(t)v(t)),\end{aligned}\right\} \quad t \neq t_k,\\ &\left.\begin{aligned}u(t_k^+) &= (1 + h_k)u(t_k),\\ v(t_k^+) &= (1 + g_k)v(t_k),\end{aligned}\right\} \quad t = t_k,\ k = 1, 2, \ldots. \end{aligned}$$

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The sufficient conditions on the uniform persistence and extinction of species are established by applying the theory of persistence of dynamical systems, asymptotically autonomous semiflows and the comparison theorem of impulsive differential equations. In [7], the authors investigated the following nonautonomous N-species Lotka–Volterra competitive system with impulsive effects

$$\begin{aligned} \dot{u}_i(t) &= u_i(t)\left[a_i(t) - \sum_{l=1}^{n} b_{il}(t)u_l(t)\right], \quad t \neq t_k,\ k = 1, 2, \ldots, \\ u_i(t_k^+) &= (1 + p_{ik})u_i(t_k), \quad i = 1, 2, \ldots, n. \end{aligned} \tag{1.1}$$

The sufficient conditions on the persistence of species and global stability of the system are established by means of the average value conditions (see [18–20]) and the methods of inequalities estimate and Liapunov functions.

However, we notice that in [7] the authors always assumed that impulsive coefficients p_{ik} in system (1.1) satisfy the following condition

$$-1 < p_{ik} \le 0, \quad i = 1, 2, \ldots, n,\ k = 1, 2, \ldots. \tag{1.2}$$

Obviously, this assumption is not always satisfied in many practical biological problems. In this paper, we will continue to investigate system (1.1) under the case which condition (1.2) does not satisfy. We rewritten system (1.1) in the following form

$$\begin{aligned} \dot{x}_i(t) &= x_i(t)\left[a_i(t) - \sum_{j=1}^{n} b_{ij}(t)x_j(t)\right], \quad t \neq t_k, \\ x_i(t_k^+) &= h_{ik}x_i(t_k), \quad i = 1, 2, \ldots, n,\ k = 1, 2, \ldots. \end{aligned} \tag{1.3}$$

For system (1.3) we always assume that $a_i(t)$ and $b_{ij}(t)$ $(i, j = 1, 2, \ldots, n)$ are bounded and continuous functions defined on $R_+ = [0, \infty)$, $b_{ij}(t) \ge 0$ for all $t \in R_+$ and impulsive coefficients h_{ik} for any $i = 1, 2, \ldots, n$ and $k = 1, 2, \ldots$ are positive constants. We will establish some new sufficient conditions on the permanence of species and global attractivity for system (1.3). We will see that in many special cases these conditions can be easily checked and reduced to some well-known results. The methods used in this paper is to use the inequalities estimate and Liapunov functions which are motivated by works on the persistence, permanence, extinction and global stability for nonautonomous Lotka–Volterra N-species competitive systems with/without impulsive effects given in [7,21].

The organization of this paper is as follows. In the next section, an important lemma on the single-species impulsive logistic systems has been introduced as a preliminary. In Section 3, the main results of this paper are stated and proved.

2. Impulsive logistic system

In this section, as a preliminary we consider the following impulsive logistic system

$$\begin{aligned} \dot{x}(t) &= x(t)(\alpha(t) - \beta(t)x(t)), \quad t \neq t_k, \\ x(t_k^+) &= h_k x(t_k),\ k = 1, 2, \ldots, \end{aligned} \tag{2.1}$$

where $\alpha(t)$ and $\beta(t)$ are bounded and continuous functions defined on R_+, $\beta(t) \ge 0$ for all $t \in R_+$ and impulsive coefficients h_k for any $k = 1, 2, \ldots$ are positive constants. We have the following result.

Lemma 2.1. *Suppose that there is a positive constant ω such that*

$$\liminf_{t\to\infty} \left(\int_t^{t+\omega} \beta(s)\mathrm{d}s\right) > 0, \tag{2.2}$$

$$\liminf_{t\to\infty} \left(\int_t^{t+\omega} \alpha(s)\mathrm{d}s + \sum_{t \le t_k < t+\omega} \ln h_k\right) > 0 \tag{2.3}$$

and function

$$h(t, \mu) = \sum_{t \le t_k < t+\mu} \ln h_k$$

is bounded on $t \in R_+$ and $\mu \in [0, \omega)$. Then we have

(a) *There exist positive constants m and M such that*

$$m \le \liminf_{t\to\infty} x(t) \le \limsup_{t\to\infty} x(t) \le M$$

for any positive solution $x(t)$ of system (2.1).

(b) $\lim_{t\to\infty}(x^{(1)}(t) - x^{(2)}(t)) = 0$ *for any two positive solutions $x^{(1)}(t)$ and $x^{(2)}(t)$ of system (2.1).*