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Global attractive periodic solutions of BAM neural networks with continuously distributed delays in the leakage terms ☆

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ABSTRACT

Bidirectional associative memory (BAM) model is considered with the introduction of continuously distributed delays in the leakage (or forgetting) terms. By using continuation theorem in coincidence degree theory and the Lyapunov functional, some very verifiable and practical algebraic mean delay dependent criteria on the existence and global attractive periodic solutions are derived.

1. Introduction

In this paper, we are concerned with the existence and global attractivity of periodic solutions for bidirectional associative memory neural networks with continuously distributed delays in the leakage terms:

$$\left.\begin{aligned} x_i'(t) &= -a_i \int_0^\infty h_i^{(1)}(s)x_i(t-s)\mathrm{d}s + \sum_{j=1}^{p} a_{ij} f_j\left(\int_0^\infty h_{ij}(s)y_j(t-s)\mathrm{d}s\right) + I_i(t) \\ y_j'(t) &= -b_j \int_0^\infty h_j^{(2)}(s)x_j(t-s)\mathrm{d}s + \sum_{i=1}^{m} b_{ji} g_i\left(\int_0^\infty l_{ji}(s)x_i(t-s)\mathrm{d}s\right) + J_j(t) \end{aligned}\right\}, \tag{1.1}$$

where

(H1) a_i, b_j, a_{ij} and b_{ji} are real constants, $a_i > 0$, $b_j > 0$, $I_i, J_j \in C(\mathbf{R}, \mathbf{R})$, I_i and J_j are ω-periodic functions, where $\omega > 0$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, p$;

(H2) $h_i^{(1)}, h_j^{(2)} \in C(\mathbf{R}^+, \mathbf{R}^+)$, $\int_0^\infty h_i^{(1)}(s)\mathrm{d}s = \int_0^\infty h_j^{(2)}(s)\mathrm{d}s = 1$, $\int_0^\infty s h_i^{(1)}(s)\mathrm{d}s < \infty$ and $\int_0^\infty s h_j^{(2)}(s)\mathrm{d}s < \infty$, $h_{ij}, l_{ji} \in C(\mathbf{R}^+, \mathbf{R}^+)$, $\int_0^\infty h_{ij}(s)\mathrm{d}s = \int_0^\infty l_{ji}(s)\mathrm{d}s = 1$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, p$, $\mathbf{R}^+ = [0, \infty)$;

(H3) There exist constants $L_j^f > 0$, $L_i^g > 0$, such that $|f_j(u) - f_j(v)| \le L_j^f |u - v|$, $|g_i(u) - g_i(v)| \le L_i^g |u - v|$ for any $u, v \in \mathbf{R}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, p$.

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A class of networks known as bidirectional associative memory (BAM) neural networks has been introduced and studied by Kosko [1–3]. Subsequently, BAM neural networks with axonal transmission delays such as

$$\left.\begin{aligned} x_i'(t) &= -a_i x_i(t) + \sum_{j=1}^{p} a_{ij} f_j(y_j(t-\sigma_j^{(2)})) + I_i, \quad i = 1, 2, \ldots, m \\ y_j'(t) &= -b_j y_j(t) + \sum_{i=1}^{m} b_{ji} g_i(x_i(t-\sigma_i^{(1)})) + J_j, \quad j = 1, 2, \ldots, p \end{aligned}\right\}, \tag{1.2}$$

and

$$\left.\begin{aligned} x_i'(t) &= -a_i x_i(t) + \sum_{j=1}^{p} a_{ij} f_j\left(\int_0^{\infty} h_{ij}(s) y_j(t-s)\mathrm{d}s\right) + I_i \\ y_j'(t) &= -b_j y_j(t) + \sum_{i=1}^{m} b_{ji} g_i\left(\int_0^{\infty} l_{ji}(s) x_i(t-s)\mathrm{d}s\right) + J_j \end{aligned}\right\}, \tag{1.3}$$

have been studied by several authors (see [4–11] and the references therein). Neural networks have been designed to solve a variety of problems; when neural networks are designed to solve optimization problems, it is expected that such networks have a unique equilibrium which is globally asymptotically or exponentially stable. For BAM neural networks with periodic coefficients and discrete delays (or continuously distributed delays), the existence and global asymptotical or exponential stability of periodic solutions also have been discussed in [12–15].

Almost all the models of the BAM are variations of the coupled systems of differential equations in which the positive constants a_i, b_j denote the timescales of the respective layers of the network; the first terms in each of the right side of (1.2) (or (1.3)) correspond to a stabilizing negative feedback of the systems which act instantaneously without time delay; these terms are variously known as forgetting or leakage terms (see for instance Kosko [1], Haykin [16]). It is known from the literature on population dynamics (see Gopalsamy [17]) that time delays in the stabilizing negative feedback terms will have a tendency to destabilize a system. Since time delays in the leakage terms are usually not easy to handle, such delays have only been considered in [18] for the following BAM neural networks

$$\left.\begin{aligned} x_i'(t) &= -a_i x_i(t-\tau_i^{(1)}) + \sum_{j=1}^{n} a_{ij} f_j(y_j(t-\sigma_j^{(2)})) + I_i, \quad i = 1, 2, \ldots, n \\ y_j'(t) &= -b_j y_j(t-\tau_j^{(2)}) + \sum_{i=1}^{n} b_{ji} g_i(x_i(t-\sigma_i^{(1)})) + J_j, \quad j = 1, 2, \ldots, n \end{aligned}\right\}. \tag{1.4}$$

By constructing the degenerate Lyapunov–Kravsovskii functional, Gopalsamy [18] has obtained some delay dependent sufficient conditions for (1.4) to have a stable equilibrium.

Motivated by the idea of [18], we consider (1.1) with the incorporation of continuously distributed delays in the leakage terms and obtain delay dependent sufficient conditions for the system to have a global periodic attractor under periodic inputs by using the coincidence degree theory and Lyapunov functionals.

2. Preliminaries

Let X and Z be real normed vector spaces let $L : \operatorname{Dom} L \subset X \to Z$ be a linear mapping, and $N : X \to Z$ is continuous mapping. The mapping L will be called a Fredholm mapping of index zero if $\dim\ker L = \operatorname{codim}\operatorname{Im} L < +\infty$ and $\operatorname{Im} L$ is closed in Z. If L is a Fredholm mapping of index zero there exist continuous projections $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Im} P = \ker L$, $\operatorname{Im} L = \ker Q = \operatorname{Im}(I - Q)$. It follows that $L|_{\operatorname{Dom} L\cap\ker P} : (I - P)X \to \operatorname{Im} L$ is invertible. We denote its inverse by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $K_P(I - Q)N : \bar{\Omega} \to X$ is compact. Since $\operatorname{Im} Q$ is isomorphic to $\ker L$, there exists isomorphism $J : \operatorname{Im} Q \to \ker L$.

Lemma 2.1 ([19]). *Let L be a Fredholm mapping of index zero, and let N be L-compact on $\bar{\Omega}$. Suppose that*

(a) $Lx \neq \lambda Nx$, $\forall x \in \operatorname{Dom} L \cap \partial\Omega$, $\lambda \in (0, 1)$;
(b) $QNx \neq 0$, $\forall x \in \ker L \cap \partial\Omega$ *and*

$$\deg\{JQN, \Omega \cap \ker L, 0\} \neq 0.$$

Then $Lx = Nx$ has at least one solution in $\operatorname{Dom} L \cap \bar{\Omega}$.

For any $x = (x_1, \ldots, x_n)^T \in \mathbf{R}^n$, its norm is defined by $|x| = \max_{1\le i\le n} |x_i|$. In the following parts of this paper, we always choose $X = Z = \{x \in C(\mathbf{R}, \mathbf{R}^n) | x(t + \omega) \equiv x(t)\}$, $\forall x \in X$, the norm of x is defined by $\|x\| = \max_{[0,\omega]} |x(t)|$, One can see X is a Banach space. Moreover, for any $z \in C(\mathbf{R}, \mathbf{R})$, $|z|_2$ is given by

$$|z|_2 = \sqrt{\int_0^{\omega} |z(t)|^2 \mathrm{d}t}.$$