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Blowup and global existence of solutions for a catalytic converter in interphase heat-transfer

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Abstract

In this paper we investigate the blowup property and global existence of a solution for a coupled system of first-order partial differential equation and ordinary differential equation which arises from a catalytic converter in automobile engineering. It is shown, in terms of a single physical parameter σ, that a unique bounded global solution exists if $\sigma < \underline{\sigma}$ and the solution blows up in finite time if $\sigma > \overline{\sigma}$, where $\underline{\sigma} < \overline{\sigma}$. Various estimates for $\overline{\sigma}$ and its associate blow-up time T^* are explicitly given. The value of T^* can be used to estimate the ignition time and ignition length of the ignition system which is an important concern in automobile engineering.

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Keywords: Blowup of solution; Global existence; Upper and lower solutions; Monotone iterations; Interphase heat-transfer; Ignition time

1. Introduction

A catalytic converter is a device located in the exhaust system of an automobile between the engine output and the exhaust tailpipe. Pollutant gases flowing out of the engine pass through it and undergo chemical processes by which they converted into relatively harmless gases. From an environment point of view how to cope with motor vehicle exhaust emission is an increasing concern in automobile engineering. This concern and many others lead to various mathematical models for the study of interphase heat-transfer problem in catalytic converter. In [4] Leighton and Chang proposed a mathematical model which governs the temperature of the vehicle exhaust emission T_g and the temperature of the catalytic converter T_s. This model consists of a first-order partial differential equation

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and an ordinary differential equation in the form:

$$\begin{cases} \pi\widetilde{a}^2(\rho c_p)_{\mathrm{g}}\left[\dfrac{\partial T_{\mathrm{g}}}{\partial t}+U\dfrac{\partial T_{\mathrm{g}}}{\partial x}\right]=2\pi\widetilde{a}h(T_{\mathrm{s}}-T_{\mathrm{g}}), & \\ 2\pi\widetilde{a}\Delta r(\rho c_p)_{\mathrm{s}}\dfrac{\partial T_{\mathrm{s}}}{\partial t}=2\pi\widetilde{a}h(T_{\mathrm{g}}-T_{\mathrm{s}})+(2\pi\widetilde{a}\Delta r+\pi\widetilde{a}^2)A\mathrm{e}^{\beta(T_{\mathrm{s}}-T_{\mathrm{g}}^{\mathrm{in}})} & t>0,\ 0<x\leqslant l; \\ T_{\mathrm{g}}(t,0)=T_{\mathrm{g}}^{\mathrm{in}} & t>0; \\ T_{\mathrm{g}}(0,x)=T_{\mathrm{s}}(0,x)=T_{\mathrm{s}}^0 & 0<x\leqslant l. \end{cases} \tag{1.0}$$

The physical meaning of the functions and parameters in this system are given as follows:

$\widetilde{a}$ is the pore radius; A is the pre-exponential factor of zeroth order kinetics; c_p is the heat capacity; β is the inverse Frank-Kamenetskii temperature; $T_{\mathrm{g}}^{\mathrm{in}}$ is the inlet temperature of the gas; T_{s}^0 is the initial temperature of the solid phases; U is a linear gas velocity; h is a heat-transfer coefficient; ρ is the gas density; Δr is the thin wall thickness; l is the length of the converter.

The main concern of [4] is to estimate the ignition time t_{ig} and the ignition length L_∞ using mechanical experimentation and numerical simulation. In the case of fast ignition (that is, Damkohler parameter χ is relatively large) the relations between t_{ig} and the blow-up time t_{ig}^∞ is given by

$$(t_{\mathrm{ig}}/t_{\mathrm{ig}}^\infty)=2.50+\chi(\ln\eta-0.34), \tag*{((1.0)$_{\mathrm{a}}$)}$$

while for low reaction rate where χ is small, this relation becomes

$$(t_{\mathrm{ig}}/t_{\mathrm{ig}}^\infty)=1+2\chi^{1/2}[\ln(\chi^{1/2}/2\eta)]^{1/2}. \tag*{((1.0)$_{\mathrm{b}}$)}$$

The ignition length is given by

$$L_\infty=U_{\mathrm{eff}}t_{\mathrm{ig}}^\infty. \tag*{((1.0)$_{\mathrm{c}}$)}$$

In the above relations, $\eta=\beta(T_{\mathrm{g}}^{\mathrm{in}}-T_{\mathrm{s}}^0)$ and U_{eff} is the effective velocity of the rod (cf. [6]). Hence to estimate the ignition time and ignition length it is crucial to find the blow-up time of the system (1.0). Although this model will break down before the actual blow-up time it provides a good estimate of the ignition time and ignition length.

To simplify the equations in (1.0) we let

$$u=\eta+\beta(T_{\mathrm{g}}-T_{\mathrm{g}}^{\mathrm{in}}),\quad v=\eta+\beta(T_{\mathrm{s}}-T_{\mathrm{g}}^{\mathrm{in}}),$$

$$a=U,\quad b=h/\Delta r(\rho c_p)_{\mathrm{s}},\quad c=2h/\widetilde{a}(\rho c_p)_{\mathrm{g}},\quad \lambda=(2\Delta r+\widetilde{a})A\beta/2\Delta r(\rho c_p)_{\mathrm{s}}\mathrm{e}^{\eta}.$$

Then problem (1.0) becomes

$$\begin{cases} \dfrac{\partial u}{\partial t}+a\dfrac{\partial u}{\partial x}+cu=cv, \quad \dfrac{\partial v}{\partial t}+bv=bu+\lambda\exp(v), & t>0,0<x\leqslant l; \\ u(t,0)=\eta, & t>0; \\ u(0,x)=u_0(x),\, v(0,x)=v_0(x), & 0<x\leqslant l. \end{cases} \tag{1.1}$$

The above model has been investigated in [1] for the existence and uniqueness of a local solution using integral representation. Also discussed was the maximal time interval for the existence of solutions but not the blow-up phenomenon of the solution. In this paper we use the method of upper and lower solutions and its associated monotone iteration to obtain sufficient conditions for the finite time blow-up property of the solution as well as the global existence of a solution. The estimate of the blow-up time from this analysis is useful for obtaining the critical ignition time and critical ignition length.

Literature dealing with catalytic converter is extensive and most of the discussions are devoted to mechanical experimentations and numerical simulations (cf. [2–6,10–12]). In [10] the authors considered a system of heat equation and ordinary equation for studying the transient behavior of a catalytic converter in the first few minutes of operation while the work of [3] is concerned with a control cost function as an optimal control problem. Other models and discussions can be found in the above references. The purpose of this paper is to present a mathematical analysis for problem (1.1) with emphasis on the finite time blow-up property of the solution. It turns out that both the blow-up of solutions and the existence of global solutions can be determined by a single parameter $\sigma\equiv\lambda/b$. Specifically, we show