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The existence and attractivity of time periodic solutions for evolution equations with delays

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Abstract

In this paper, we investigate the existence and global attractivity of time periodic solutions for evolution equations with several delays under some reasonable assumptions. The key-steps are constructing some suitable Lyapunov functionals and establishing the prior bound for all possible periodic solutions. Some illustrative examples are presented in the last section.

MSC: 34K13; 35K57

Keywords: Time periodic solutions; Global attractivity; Uniformly asymptotic stability; Prior bound

1. Introduction

The existence or attractivity of periodic solutions for evolution equations with delays has been considered in several works, see for examples [1,3–11,15] and references listed therein. Most of these results are established by applying semigroup theory [3–5,7,15], Leray–Schauder continuation theorem [7,15], coincidence degree theory [15] and so on. In [7], Hino et al. investigated the existence of (almost) periodic solutions for damped wave equations by using the key assumption that there exists a bounded solution. In [1], Burton and Zhang obtained the time periodic solutions to some evolution equations with infinity delay by mean of Granas's fixed point theorem. The theory of partial differential equations with delay(s) has seen considerable development, see the monographs of Wu [15] and Hale [3,4], where numerous properties of their solutions are studied. For the more special work, we can read the references therein [3,5,12,15].

In this paper, we investigate the existence and attractivity of time periodic solutions for the following evolution equations with delays:

$$\begin{cases} \dfrac{\partial}{\partial t}u(t,x) = \dfrac{\partial^2}{\partial x^2}u(t,x) + au(t,x) + f(u(t-\tau_1,x),\ldots,u(t-\tau_n,x)) + g(t,x), & 0<x<1,\quad t>0,\\ u(t,0)=u(t,1)=0, & t>0. \end{cases} \tag{1.1}$$

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This type of equations is usually used to model some process of biology, such as, if $n = 1$ and $f(r) = e^{-kr}(k > 0)$, (1.1) is named as Hematopoiesis model, if $n = 1$ and $f(r) = \alpha r/(1 + r^{\beta})(\alpha, \beta > 0)$, (1.1) is called blood cell production model [14] and if $n = 1$ and $f(r) = re^{-kr}(k > 0)$, (1.1) is the Nicholson's blowflies model [2,13].

Let $H = L^2(0, 1)$, with the norm $\|\cdot\|$ and scalar inner product $\langle\cdot,\cdot\rangle$. Let $A = -\triangle$ (the Laplace operator) with domain $D(A) = H_0^1 \cap H^2$. Firstly, we consider the evolution equation in $L^2(0, 1)$

$$u'(t) + Au(t) = \lambda G(t, u_t), \quad \lambda \in (0, 1), \tag{1.2}$$

where $u_t \in C := C([-\tau, 0], H)$, $G : R \times C \to H$, $\tau = \max\{\tau_i : i = 1, 2, \ldots, n\}$ and $u_t(\theta) = u(t + \theta)$ for $\theta \in [-\tau, 0]$. Furthermore, if $G(t + T, \varphi) = G(t, \varphi)$ for any $\varphi \in C$ and some $T > 0$, in line with the similar arguments in [10, Theorem 3.1], we would achieve the following result.

Theorem 1.1. *Suppose the following conditions hold*:

(1) *there exists $D > 0$ such that if $u(t)$ is a T-periodic solution of* (1.2) *for some $\lambda \in (0, 1)$, then* $\sup_{t\in[0,T]}\|u(t)\|_{H^1} < D$;
(2) *$G : [0, T] \times C \to H$ is continuous and takes bounded sets into bounded sets. Then* (1.2) *has a T-periodic solution for $\lambda = 1$.*

The paper is organized as follows. In Section 2, we will establish the existence and attractivity of the time periodic solutions for some evolution equations. Section 3 is devoted to some examples that show the applicability of our results.

2. Existence and attractivity of time periodic solutions

In this section, firstly, we consider the existence and attractivity of periodic solutions for (1.1). We assume that there exist some positive constants β_i, K and T satisfying following conditions.

(A_1) The integer $n \leqslant 3$;
(A_2) $g \neq 0$, $g(t + T, x) = g(t, x)$, $f(0, \ldots, 0) = 0$ and

$$|f(r_1, \ldots, r_n) + g(t, x)| \leqslant \sum_{i=1}^{n} \beta_i |r_i| + K;$$

(A_3) $(|a| + 2)^2 + \sum_{i=1}^{n} \beta_i^2 < \pi^2 + 1$;
(A_4) $|f(x_1, \ldots, x_n) - f(y_1, \ldots, y_n)| \leqslant \sum_{i=1}^{n} \beta_i |x_i - y_i|$.

We consider the homotopy equation of (1.1) (for some $\lambda \in (0, 1)$)

$$u'_t = u_{xx} + \lambda(au(t, x) + f(u(t - \tau_1, x), \ldots, u(t - \tau_n, x)) + g(t, x)), \tag{2.1}$$

where $u'_t := (\partial/\partial t)u(t, x)$.

Lemma 2.1. *Suppose that* (A_1), (A_2) *and* (A_3) *hold. Then there is a constant $C_1 > 0$ such that each T-periodic solution $u(t)$ of* (2.1) *satisfies*

$$\sup_{0\leqslant t\leqslant T} \int_0^1 [u_x^2(t, x) + u^2(t, x)]\,dx \leqslant C_1.$$

Proof. Since

$$(|a| + 2)^2 + \sum_{i=1}^{n} \beta_i^2 < \pi^2 + 1,$$