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Bifurcations and chaos of coupled electrical circuits

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Abstract

The dynamics of two coupled chaotic electrical circuits has been investigated in this paper. Transition boundaries have been obtained to divide the parameter space into regions associated with different types of phase portraits. It is pointed out that two stable equilibrium points coexist for certain parameter conditions, which may evolve to different chaotic attractors via period-doubling bifurcations, respectively.

Keywords: Coupled electrical circuits; Bifurcation; Chaotic attractor; Transition boundary

1. Introduction

The dynamics of nonlinear electrical circuits has been widely investigated during the last decade [6,7], since very complicated behaviors can be observed in a lot of electrical circuits [9], for example, chaos in Chua's circuits [4]. For most of the cases, ordinary differential equations can be established to model the real circuits and bifurcation details have been presented with the variation of parameters, which can be verified by numerical simulations as well as experimental results [5,14]. Fang and Tong presented a new approach to the measurement of parameters in an extremely unstable chaotic system of chaotic circuits [8]. Yu and Leung introduced the notion called simplest normal form to determine bifurcation of a nonlinear electrical circuit [15]. For coupled oscillators, because there may exist internal resonance, many interesting phenomena, such as energy saturation, can be observed [11]. The concept of synchronization has been introduced to understand the dynamics of coupled oscillators [12]. Specially, phase synchronization due to weak interactions between two chaotic systems has been revealed although the oscillators remain chaotic [13]. Based on Lyapunov stability theory, Agiza and Matouk derived sufficient conditions for the synchronization between the original Chua's circuit and the modified Chua's circuit [1]. In our recent papers [2,3], we explored the dynamics of coupled van der Pol oscillators and found that two cascading of period-doubling bifurcations may join together to yield modulated chaos. Furthermore, phase plane analysis has been introduced to explore the bifurcation details of different equation [16,17].

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In this paper, we consider the following two coupled chaotic electrical circuits, written as

$$\dddot{x} = a\ddot{x} - c\dot{x} + (|x| - 1) + \varepsilon(x - y), \tag{1}$$
$$\dddot{y} = b\ddot{y} - c\dot{y} + y - y^3 + \varepsilon(y - x), \tag{2}$$

which can be derived from two real electrical circuits with linear interaction [10]. We focus on the influence of the parameters as well as coupled terms on the dynamical behaviors of the system and try to explore the bifurcation details and chaotic phenomena in the evolution of the coupled vector field.

2. Bifurcation analysis

We first consider the two electrical circuits without coupling, i.e. $\varepsilon = 0$, respectively. For Eq. (1), the equilibrium points can be described as $E_{11} = (x, \dot{x}, \ddot{x}) = (-1, 0, 0)$, $E_{12} = (x, \dot{x}, \ddot{x}) = (-1, 0, 0)$, the stabilities of which can be determined by the characteristic polynomials associated, expressed in the form

$$\begin{aligned} &\lambda^3 - a\lambda^2 + c\lambda + 1 = 0 \quad \text{for } E_{11}, \\ &\lambda^3 - a\lambda^2 + c\lambda - 1 = 0 \quad \text{for } E_{12}, \end{aligned} \tag{3}$$

respectively.

It is obvious to see that E_{12} is always unstable and no zero eigenvalue can be obtained from the characteristic equations, implying no simple bifurcation can be observed. The stability condition for E_{11} can be expressed by $a < 0$, $ac + 1 < 0$, which yields the following bifurcation set:

$$H: \; ac + 1 = 0 \quad (ac < 0), \tag{4}$$

on which the Hopf bifurcation may take place.

The bifurcation set is plotted in Fig. 1, which divides the parameter plane $(a - c)$ into two regions associated with different types of phase trajectories. In Region ①, only stable equilibrium point E_{11} can be obtained (see Fig. 2a). E_{11} loses the stability on the Hopf bifurcation set H, which yields a limit cycle in Region ② (see Fig. 2b).

For fixed parameter $c = 1.0$, the bifurcation diagram with the variation of parameter a has been plotted in Fig. 3. It can be observed that a cascading of period-doubling bifurcations leads the system to chaos (see Fig. 4). Periodic solution can be observed in the chaotic region (see Fig. 4e), which may quickly lose stability and result in chaotic solution (see Fig. 4f).

With further increase of the parameter a, the chaotic solution finally settles down to a period-2 movement (see Fig. 5). All the trajectories tend to infinite as $a > -0.545066$, implying no bounded solution can be obtained.

Now we turn to the second equation (2) for $\varepsilon = 0$. Three fixed equilibrium points, denoted by $E_{21} = (y, \dot{y}, \ddot{y}) = (-1, 0, 0)$, $E_{22} = (y, \dot{y}, \ddot{y}) = (0, 0, 0)$, $E_{23} = (y, \dot{y}, \ddot{y}) = (1, 0, 0)$, can be obtained, the stabilities of which can be

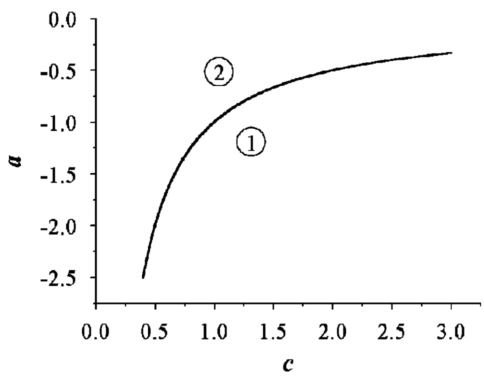

Fig. 1. Bifurcation set on parameter plane $(a - c)$.