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Existence of periodic solutions in predator–prey and competition dynamic systems☆

Martin Bohner[a,*], Meng Fan[b], Jimin Zhang[b]

[a] Department of Mathematics and Statistics, University of Missouri–Rolla, Rolla, MO 65401, USA
[b] School of Mathematics and Statistics, and Key Laboratory for Vegetation Ecology, Northeast Normal University, Changchun, Jilin 130024, PR China

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Abstract

In this paper, we systematically explore the periodicity of some dynamic equations on time scales, which incorporate as special cases many population models (e.g., predator–prey systems and competition systems) in mathematical biology governed by differential equations and difference equations. Easily verifiable sufficient criteria are established for the existence of periodic solutions of such dynamic equations, which generalize many known results for continuous and discrete population models when the time scale $\mathbb{T}$ is chosen as $\mathbb{R}$ or $\mathbb{Z}$, respectively. The main approach is based on a continuation theorem in coincidence degree theory, which has been extensively applied in studying existence problems in differential equations and difference equations but rarely applied in dynamic equations on time scales. This study shows that it is unnecessary to explore the existence of periodic solutions of continuous and discrete population models in separate ways. One can unify such studies in the sense of dynamic equations on general time scales.

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1. Introduction

In the past decades, mathematical ecology has seen much progress, especially in population dynamics. Most natural environments are physically highly variable, and in response, birth rates, death rates, and other vital rates of populations, vary greatly in time. Theoretical evidence to date suggests that many population and community patterns represent intricate interactions between biology and variation in the physical environment (see [4] and other papers in the same issue). Therefore, the focus in theoretical models of population and community dynamics must be not only on how populations depend on their own population densities or the population densities of other organisms, but also on how

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* Corresponding author. Tel.: +1 573 341 6208; fax: +1 573 341 4741.
E-mail addresses: bohner@umr.edu (M. Bohner), mfan@nenu.edu.cn (M. Fan).

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populations change in response to the physical environment. When the environmental fluctuation is taken into account, a model must be nonautonomous, and hence, of course, more difficult to analyze in general. But, in doing so, one can and should also take advantage of the properties of those varying parameters. For example, one may assume the parameters are periodic or almost periodic for seasonal reasons. Due to the recognition that temporal fluctuations in the physical environment are a major driver of population fluctuations, there has been more and more theoretical attention to predict the characteristic of the resultant population fluctuations.

A very basic and important problem in the study of a population growth model with a periodic environment is the global existence and stability of a positive periodic solution, which plays a similar rôle as a globally stable equilibrium does in an autonomous model. Thus, it is reasonable to seek conditions under which the resulting periodic nonautonomous system would have a positive periodic solution that is globally asymptotically stable. Much progress has been seen in this direction. Careful investigation reveals that it is similar to explore the existence of periodic solutions for nonautonomous population models governed by ordinary differential equations and their discrete analogue in the approaches, the methods and the main results. For example, extensive research reveals that many results concerning the existence of periodic solutions of predator–prey systems modelled by differential equations can be carried over to their discrete analogues based on the coincidence theory, for example, [5,7,9–11,14,15,18,19]. It is natural to ask whether we can explore such an existence problem in a unified way.

The theory of calculus on time scales (see [2,3] and references cited therein) was initiated by Stefan Hilger in his Ph.D. Thesis in 1988 [13] in order to unify continuous and discrete analysis, and it has a tremendous potential for applications and has recently received much attention since his foundational work. It has been created in order to unify the study of differential and difference equations. Many results concerning differential equations carry over quite easily to corresponding results for difference equations, while other results seem to be completely different from their continuous counterparts. The study of dynamic equations on general time scales can reveal such discrepancies and help avoid proving results twice—once for differential equations and once again for difference equations. The two main features of the calculus on time scales are unification and extension. To prove a result for a dynamic equation on a time scale is not only related to the set of real numbers or set of integers but those pertaining to more general time scales.

The principle aim of this paper is to systematically unify the existence of periodic solutions of population models modelled by ordinary differential equations and their discrete analogues in form of difference equations and to extend these results to more general time scales. The approach is based on a continuation theorem in coincidence degree, which has been widely applied to deal with the existence of periodic solutions of differential equations and difference equations. This paper is the first one to apply coincidence degree theory to explore the existence of periodic solutions of dynamic equations on time scales. The setup of this paper is as follows. In the coming section, we present some preliminary results such as the calculus on time scales and the continuation theorem in coincidence degree theory. Then we systematically explore the existence of periodic solutions of dynamic equations on time scales of predator–prey type and competition type. This study reveals that, when we deal with the existence of positive periodic solutions of population models, it is unnecessary to prove results for differential equations and separately again for difference equations. One can unify such problems in the frame of dynamic equations on time scales.

2. Preliminaries

In this section, we briefly give some elements of the time scales calculus, recall the continuation theorem from coincidence degree theory, and prove an auxiliary result that is needed in the paper.

First, let us present some foundational definitions and results from the calculus on time scales so that the paper is self-contained. For more details, one can see [2,3,13].

Notation 2.1. Throughout this paper, the symbol $\mathbb{T}$ denotes a *time scale*, i.e., an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. Let $\omega > 0$. Throughout, the time scale $\mathbb{T}$ is assumed to be *ω-periodic*, i.e., $t \in \mathbb{T}$ implies $t + \omega \in \mathbb{T}$. In particular, the time scale $\mathbb{T}$ under consideration is unbounded above and below. Some examples of such time scales are

$$\mathbb{R}, \quad \mathbb{Z}, \quad \bigcup_{k\in\mathbb{Z}} [2k, 2k+1], \quad \bigcup_{k\in\mathbb{Z}} \bigcup_{n\in\mathbb{N}} \left\{ k + \frac{1}{n} \right\}.$$