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Pseudo-almost periodic solutions for non-autonomous neutral differential equations with unbounded delay

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Abstract

In this note we establish the existence of pseudo-almost periodic solutions for a non-autonomous partial neutral functional differential with unbounded delay. We apply our abstract results to establish the existence of this type of solutions for a neutral differential equation which arises in the theory of heat conduction in materials with fading memory.
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1. Introduction

In this paper we study the existence of *pseudo*-almost periodic solutions for a class of non-autonomous first-order abstract partial neutral functional differential equations with unbounded delay described in the form

$$\frac{\mathrm{d}}{\mathrm{d}t} D(t, u_t) = A(t)D(t, u_t) + g(t, u_t), \tag{1.1}$$

where $A(t) : \mathscr{D} \subset \mathbb{X} \to \mathbb{X}$ is a family of densely defined closed linear operators; $(\mathbb{X}, \|\cdot\|)$ is a Banach space; the history $u_t : (-\infty, 0] \to \mathbb{X}$, $u_t(\theta) := u(t+\theta)$, belongs to some abstract phase space $\mathscr{B}$ defined axiomatically; $D(t, \psi) = \psi(0) + f(t, \psi)$ and $f, g : \mathbb{R} \times \mathscr{B} \to \mathbb{X}$ are suitable functions.

Partial neutral integro-differential equation with unbounded delay arises, for instance, in the theory development in Gurtin and Pipkin [9] and Nunziato [18] for the description of heat conduction in materials with fading memory. In the classic theory of heat conduction, it is assumed that the internal energy and the heat flux dependent linearly on the temperature $u(\cdot)$ and on its gradient $\nabla u(\cdot)$. Under these conditions, the classic heat equation describes sufficiently well the evolution of the temperature in different type of materials. However, this description is not satisfactory in materials with fading memory. In the theory developed in [9,18], the internal energy and the heat flux are described as functionals

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of u and u_x. The next system, see for instance [3,17,20], has been frequently used to describe this phenomena,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[c_0 u(t,x) + \int_{-\infty}^{t} k_1(t-s)u(s,x)\,\mathrm{d}s\right] = c_1 \triangle u(t,x) + \int_{-\infty}^{t} k_2(t-s)\triangle u(s,x)\,\mathrm{d}s,$$

$$u(t,x) = 0, \quad x \in \partial\Omega.$$

In this system, $\Omega \subset \mathbb{R}^n$ is open, bounded and with smooth boundary; $(t,x) \in [0,\infty) \times \Omega$; $u(t,x)$ represents the temperature in x at the time t; c_1, c_2 are physical constants and $k_i : \mathbb{R} \to \mathbb{R}$, $i = 1, 2$, are the internal energy and the heat flux relaxation, respectively. By assuming that the solution $u(\cdot)$ is known on $(-\infty, 0]$ and that $k_1 = k_2$, we can transform this system into an abstract neutral system with unbounded delay.

The literature relative to ordinary neutral differential equations is extensive; for more on this topic and related applications we refer the reader to Hale [11] and the references therein. Similarly, for more on abstract partial neutral functional differential equations we refer to Adimy and Ezzinbi [1], Hale [10], Wu and Xia [22] and Wu [21], for finite delay equations, and to Hernández and Henríquez [13] and Hernández [12] for equations with unbounded delay.

The existence of almost periodic, asymptotically almost periodic, pseudo-almost periodic, almost automorphic, and asymptotically almost automorphic solutions are among the most attractive topics in qualitative theory of differential equations due to their significance and applications in physics, mathematical biology, control theory, and others. The concept of the pseudo-almost periodicity, which is the central question in this paper was introduced in the literature in the early 90s by Zhang [25–28] as a natural generalization of the well-known (Bohr) almost periodicity. This new concept is then welcome to implement another existing generalization of the (Bohr) almost periodicity, the notion of asymptotically almost periodicity due to Fréchet, see, e.g. [23].

Various contributions on pseudo-almost periodic solutions to ordinary differential equations, abstract partial differential equations and abstract partial functional differential equations have recently been made, see for instance [2,4–8,16] among others. The existence and uniqueness of pseudo-almost periodic solutions to neutral equations with bounded delay described in the form

$$\frac{\mathrm{d}}{\mathrm{d}t}(u(t) + f(t,u_t)) = Au(t) + g(t,u_t),$$

has recently been obtained in [7]. For that, the first task there consisted of introducing some new spaces called pseudo-almost periodic functions of class p, which are heavily linked with the presence of (finite) delay. Next, some composition results of pseudo-almost periodic functions of class p were established. And finally, the previous results, were, subsequently utilized to obtain some existence results for (1.1) in the case when $A(t) = A$ and $\mathscr{B} = C([-p, 0], \mathbb{X})$.

To the best of our knowledge, the problem of the existence of pseudo-almost periodic solutions to " non-autonomous" functional–differential equations with "unbounded delay," in particular, abstract neutral differential equations with unbounded delay, is an untreated topic in the literature, and this is the main motivation of the present paper.

2. Preliminaries

In what follows we recall some definitions, notations and properties needed in the sequel. In particular, to deal with infinite delay we need to introduce some new classes of pseudo-almost periodic functions.

Let $(\mathbb{Z}, \|\cdot\|_{\mathbb{Z}})$ and $(\mathbb{W}, \|\cdot\|_{\mathbb{W}})$ be Banach spaces. The notation $\mathscr{L}(\mathbb{Z}, \mathbb{W})$ stands for the Banach space of bounded linear operators from $\mathbb{Z}$ into $\mathbb{W}$ equipped with its natural topology; in particular, this is simply denoted by $\mathscr{L}(\mathbb{Z})$ when $\mathbb{Z} = \mathbb{W}$. Throughout the paper, $BC(\mathbb{W}, \mathbb{Z})$ denote the collection of all bounded continuous functions from $\mathbb{W}$ into $\mathbb{Z}$ equipped with the sup norm defined by $\|u\|_\infty := \sup_{w\in\mathbb{W}} \|u(w)\|_{\mathbb{Z}}$.

In this paper, $\{A(t) : t \in \mathbb{R}\}$ is a family of closed linear operators defined on a common domain $\mathscr{D}$, which is independent of t, and dense in $\mathbb{X}$. We always suppose that the system

$$\begin{cases} u'(t) = A(t)u(t), & t \geqslant s, \\ u(s) = x \in \mathbb{X}, \end{cases} \tag{2.1}$$

has an associated evolution family of operators $\{U(t,s) : t \geqslant s \text{ with } t, s \in \mathbb{R}\}$, which is uniformly asymptotically stable.

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