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ScienceDirect
Nonlinear Analysis: Real World Applications 9 (2008) 490–499
Nonlinear Analysis
Real World Applications
www.elsevier.com/locate/na

Persistence for Lotka–Volterra patch-system with time delay

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Received 3 November 2006; accepted 17 November 2006

Abstract

In this paper, we consider the nonautonomous Lotka–Volterra system with dispersion. Under the assumption that the intrinsic growth rates of the species may be negative, we show that certain average conditions imply the uniform persistence of all species. Some known result is improved.

Keywords: Patch-system; Time delay; Uniform persistence

1. Introduction and auxiliary lemmas

One of the most basic and important problems in mathematical biology concerns the survival of species in Lotka–Volterra patch-system. For general ordinary differential equations, Levin [3] first established this kind of model for autonomous Lotka–Volterra system. Kishimoto [2] and Takeuchi [7] also studied these kinds of models, but all the coefficients in the system they studied are constants. Zeng and Chen [8] extended the autonomous Lotka–Volterra system, but still without time delay, and investigated persistence of the populations and periodic behaviour of the system. Zhang and Chen [9] extended the system in [8] to the system with time delay and also investigated persistence of the populations. For other related work, we refer to [4–6]. However, in the above papers, all the coefficients in the system they studied are nonnegative. So in this paper, we extend the system in [9] to the system which admits the intrinsic growth rates of the species to be negative. Our main focus is persistence of the population in two patches.

In this paper, we consider the following Lotka–Volterra system with dispersion:

$$\begin{cases} \dfrac{\mathrm{d}x_1(t)}{\mathrm{d}t} = x_1(t)[r_1(t) - a_1(t)x_1(t) - b_1(t)y(t)] + D_1(t)[x_2(t) - x_1(t)], \\ \dfrac{\mathrm{d}x_2(t)}{\mathrm{d}t} = x_2(t)[r_2(t) - a_2(t)x_2(t)] + D_2(t)[x_1(t) - x_2(t)], \\ \dfrac{\mathrm{d}y(t)}{\mathrm{d}t} = y(t)[r_3(t) - a_3(t)x_1(t) - b_3(t)y(t) - \beta(t)\int_{-\tau}^{0} K(s)y(t+s)\,\mathrm{d}s], \end{cases} \tag{1.1}$$

where $x_1(t)$ and $y(t)$ are the densities of species x and y in patch 1, and $x_2(t)$ is the density of species x in patch 2. Species y is confined to patch 1, while species x can diffuse between two patches. $D_i(t)$ $(i = 1, 2)$ are diffusion coefficients of species x.

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1468-1218/$ - see front matter
doi:10.1016/j.nonrwa.2006.11.014

Denote $C^+_{[t_0-\tau,t_0]} = \{\varphi = (\varphi^{(1)}, \varphi^{(2)}, \varphi^{(3)}) | \varphi^{(i)} : [t_0 - \tau, t_0] \mapsto R^+$ is a continuous function with $\varphi^{(i)}(t_0) > 0, i = 1, 2, 3\}$. For any $\varphi \in C^+_{[t_0-\tau,t_0]}$, $Z(t, \varphi) = (x_1(t, \varphi), x_2(t, \varphi), y(t, \varphi))$ is the solution of system (1.1) with the following initial conditions:

$$\begin{cases} x_i(s) = \varphi^{(i)}(s) \geqslant 0, & s \in [t_0 - \tau, t_0], \quad \varphi^{(i)}(t_0) > 0, \quad i = 1, 2, \\ y(s) = \varphi^{(3)}(s) \geqslant 0, & s \in [t_0 - \tau, t_0], \quad \varphi^{(3)}(t_0) > 0. \end{cases} \tag{1.2}$$

Now for a continuous and bounded function $f(t)$, we let $f^l = \inf_{t \geqslant t_0} f(t)$, $f^u = \sup_{t \geqslant t_0} f(t)$. And if $0 < f^l < f(t) < f^u < +\infty$, we say $f(t)$ is a strictly positive function.

In system (1.1), we always assume:

(H_1) $r_i(t)$, $a_i(t)$ $(i = 1, 2, 3)$ and $b_3(t)$ are continuous and bounded functions defined on R, which satisfy

$$a_i(t) \geqslant 0, \quad i = 1, 2, 3, \quad b_3(t) \geqslant 0,$$

and there exist positive constants ω and δ such that for any $t \in R$,

$$\int_t^{t+\omega} r_3(s)\,\mathrm{d}s \geqslant \delta, \quad \int_t^{t+\omega} b_3(s)\,\mathrm{d}s \geqslant \delta, \tag{1.3}$$

$$\int_t^{t+\omega} \bar{r}(s)\,\mathrm{d}s \geqslant \delta, \quad \int_t^{t+\omega} a(s)\,\mathrm{d}s \geqslant \delta, \tag{1.4}$$

where $\bar{r}(t) = \max\{r_1(t), r_2(t)\}$, $a(t) = \min\{a_1(t), a_2(t)\}$;

(H_2) $b_1(t)$, $\beta(t)$ and $D_i(t)$ $(i = 1, 2)$ are nonnegative, continuous and bounded functions defined on R;

(H_3) $K(s) \geqslant 0$ on $[-\tau, 0]$, $(0 \leqslant \tau < +\infty)$; and $K(s)$ is a piecewise continuous and normalized function such that $\int_{-\tau}^0 K(s)\,\mathrm{d}s = 1$.

Let $z(t) = (x_1(t), x_2(t), y(t))$ denote the solution of system (1.1) corresponding to the initial conditions (1.2).

Definition. System (1.1) is said to be uniformly persistent if there exists a compact region $K_1 \subset \operatorname{int} R^3_+$ such that every solution $z(t)$ of (1.1) eventually enters and remains in the region K_1.

The following three lemmas will be used repeatedly in the proofs of our results. The proof of the first lemma is straightforward and will be omitted.

Lemma 1.1. *Every solution $z(t)$ of* (1.1) *with the initial conditions* (1.2) *exists on the interval* $[t_0, +\infty)$ *and remains positive for all $t \geqslant t_0$.*

In fact, such solution of (1.1) are called positive solutions. Hence, in the rest of this paper, we always assume that $\varphi \in C^+_{[t_0-\tau,t_0]}$.

Consider the following equation:

$$\frac{\mathrm{d}W(t)}{\mathrm{d}t} = W(t)[A(t) - B(t)W(t)], \tag{1.5}$$

we have the following lemma.

Lemma 1.2. *Suppose that $A(t)$ and $B(t)$ are continuous and bounded functions on R, which satisfy $B(t) \geqslant 0$ and there are positive constants ω and δ, such that for any $t \in R$,*

$$\int_t^{t+\omega} A(s)\,\mathrm{d}s \geqslant \delta, \quad \int_t^{t+\omega} B(s)\,\mathrm{d}s \geqslant \delta.$$