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ScienceDirect
Nonlinear Analysis: Real World Applications 9 (2008) 547–561
Nonlinear Analysis
Real World Applications
www.elsevier.com/locate/na

The dynamics of an age structured predator–prey model with disturbing pulse and time delays ☆

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Received 25 September 2006; accepted 14 December 2006

Abstract

Many of the existing predator–prey models on stage structured populations are some ordinary differential equations (ODE) or models without a disturbing effect of human behavior. In reality, death of the juvenile during its immature stage and catching or poisoning for the prey or predator occur continuously. From this basic standpoint, we formulate a general and robust prey-dependent consumption predator–prey model with periodic harvesting (catching or poisoning) for the prey and stage structure for the predator with constant maturation time delay (through-stage time delay) and perform a systematic mathematical and ecological study. We show that the conditions for global attractivity of the 'predator-extinction' ('predator-eradication') periodic solution and permanence of the population of the model depend on time delay, so, we call it "profitless". We also show that constant maturation time delay and impulsive catching or poisoning for the prey can bring great effects on the dynamics of system by numerical analysis. In this paper, the main feature is that we introduce time delay and pulse into the predator–prey (natural enemy–pest) model with age structure, exhibit a new modeling method which is applied to investigate impulsive delay differential equations, and give some reasonable suggestions for pest management.

Keywords: Permanence; Impulsive effect; Predator–prey model with age structure; Maturation time delay; Global attractivity

1. Introduction

Effectively controlling pests has become an increasingly complex issue over the past two decades. A wide range of pest control ways are available to farmers such as biological, cultural, physical and chemical tools. Farmers often use relatively simple techniques to control the increase in insect pest amounts regardless of the balance of nature. For example, farmers often catch pests by mechanical tools or poison pests by the overuse of pesticides. However, on the one hand, overuse of chemicals have created many ecological and sociological problems, hence chemical control now needs to be used reasonably. On the other hand, eradication for pests is difficult both practically and economically as pests can breed quickly. Therefore, it may be the best way of ensuring that pest populations do not fluctuate widely from one year to the next.

☆ This work is supported by National Natural Science Foundation of China (10471117) and Natural Science Foundation of SDUST (05g016).
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1468-1218/$ - see front matter
doi:10.1016/j.nonrwa.2006.12.001

Most pests have their natural enemies, and these natural enemies can effectively suppress pests sometimes. When people catch or poison pests, pests and their natural enemies, which does die out earlier? For example, field mouses and owls lived in the northwest plain of China in 1998. The owls (natural enemy) became extinct earlier than field mouses, because the prey (pest) is catched or poisoned largely, subsequently, the survivor (field mouse) increased rapidly and overran ultimately, why? We would like to answer the question, then we need to investigate the effect of the harvesting for pests on its natural enemies. In practice, from the principle of ecosystem balance, we need only to control the pest population under the economic threshold level (ETL) and not to eradicate natural enemy totally, and hope pest population and natural enemy population can coexist when the pests do not bring about immense economic losses. In this paper, according to the above ecological background, we consider the prey-dependent consumption predator–prey (natural enemy–pest) models with age structure for the predator, the prey is catched or poisoned impulsively, details can be seen in Sections 2 and 5. In Section 3, We prove that, when the impulsive period is no longer than some threshold, i.e. $T < (1/r)\ \ln(K-(1-\delta)p^{-1}(d_2 e^{d_1\tau}/\lambda\beta))/((1-\delta)(K-p^{-1}(d_2 e^{d_1\tau}/\lambda\beta)))$ or partial destruction to pests (preys) by catching or pesticides is at a appropriate extent, i.e. $1-(Ke^{-rT})/(K-(1-e^{-rT})p^{-1}(d_2 e^{d_1\tau}/\lambda\beta)) < \delta < 1-e^{-rT}$, the predator-eradication periodic solution is globally attractive, or say, the predator population can be eradicated totally when the pest population is catched or poisoned at a certain extent. However, from the point of ecological balance and saving resources, we only need to control the pest population under the ETL in order not to eradicate predator (natural enemy) totally, so in Section 4, we further prove that, when the impulsive period is longer than the threshold, i.e., $T > -(1/r)\ \ln(1-(\delta K/K-p^{-1}(d_2 e^{d_1\tau}/\lambda\beta)))$ or partial destruction to pests (preys) by catching or pesticides is smaller than the threshold, i.e., $\delta < (1-e^{-rT})(1-(p^{-1}(d_2 e^{d_1\tau}/\lambda\beta))/K)$, pest population and natural enemy population can coexist and the system is uniformly permanent.

The predator–prey models with age structure for the predator were introduced or investigated by Hastings, Wang and Hui [4–7,21]. Since the immature predator takes τ (which is called maturation time delay) units of time to mature, the death toll during the juvenile period should be considered, so, time delays have important biological meanings in age-structured models. Hence many age-structured models with time delay were extensively studied (see [6,21,4,13,14,18,16,22,1]). In recent years, impulsive systems are found in many domains of applied sciences (see [7,19,3,11,17,20,12,24]). The investigation of impulsive delay differential equations is inchoate, and impulsive delay differential equations are almost analyzed in theory (see [23,10,15]). Time delay and impulse are introduced into predator–prey models with stage structure, which greatly enriches biologic background, but the system become nonautonomous and quite complicated, which causes us greatly difficulties to study the model. Therefore, the literature on global qualitative analysis for delay stage-structured models with impulse effect has never been seen by now. In present paper, we propose a new delay predator–prey model with age structure and impulsive effect and method which is applied to study impulsive delay differential equations.

2. Model and preliminaries

The predator–prey model with age structure for the predator was introduced by Hastings [5,6]:

$$\dot{x} = rx(1-x) - \beta x y_2,$$
$$\dot{y}_1 = \lambda\beta x y_2 - (d+m)y_1,$$
$$\dot{y}_2 = m y_1 - d y_2,$$

where $x(t)$, $y_1(t)$, $y_2(t)$ represent the densities of prey, immature and mature predator, respectively, r is the logistic intrinsic growth rate of the prey in the absence of the predator, β is the predation rate of predator and λ represents the conversion rate at which ingested prey in excess of what is needed for maintenance is translated into predator population increase, d is the death rate of predator, the maturity rate is m, which determines the mean length of the juvenile period, h is the handling time of predator. By calculation, the system has two unstable equilibria $E_0(0,0,0)$ and $E_1(1,0,0)$; when $d^2+dm-m\lambda\beta<0$, there exists a positive equilibrium $E^*(1-(\beta/r)y_2^*, y_2^*, (d/m)y_2^*)$, where $y_2^* = (m\lambda\beta - d^2 - dm)r/(m\lambda\beta^2)$.

The basic model that we consider is the one based on the idea that predator may consume an increasingly smaller proportion of killed prey as prey density increases. To investigate the effect of the harvesting for pests on its