Nonlinear Analysis: Real World Applications 10 (2009) 2939–2944

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

Energy storage and geo-massif fracture

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ARTICLE INFO

Article history:
Received 29 May 2007
Accepted 29 September 2008

Keywords:
Fracture
Earthquake
Rupture propagation
Stored energy

ABSTRACT

Since a defective body does not satisfy the condition of compatibility, the intrinsic metric of continuum with the field of defects is non-Riemannian. This causes the problem of energy flux determination in a real inhomogeneous deformed body. The energy flux is the key parameter of geomechanics, the continuum of which is essentially non-uniform.

We consider the intrinsic metric of the deformed continuum as a Finslerian $\mathbb{E}^3 \times \mathbb{E}^3$ with the metric tensor $g_{ij}(x, \dot{x}) = g_{ij}(x, \xi)$.

Taking into account the main properties of the Finsler space, we can find the metric as a Hamiltonian function. We can build the continuous field of the rays propagation directions because the energy of a deformed body is arbitrary. The continuity is very important from the physical point of view because the real energy flux starts only on the source and stops on the sink.

The energy flux can be evaluated through the Umov–Pointing vector. The direction of this vector can be received as a normal to the equienergetic surface. For the arbitrary metric, this surface is the indicatrix. Taking into account non-symmetry of orthogonality in the Finsler space, we can determine the excess of storage energy.

1. Introduction

Defining scaling characteristics of mountain rock fracture or earthquake processes is a popular general topic today. At the same time, a detailed theory of mechanical behaviour of geological massifs in linear approximation does not produce sufficiently satisfactory results, since the real continuum of geomechanics is essentially non-linear [1,2] and non-uniform. Taking into account the scale difference of geo-massif and defects, the model of a real massif is equal to the model of solids with microstructure in engineering scale. On the other hand, the behaviour of geological structures under external influences is critically important. This is related to colossal scales of possible implications – both human and material casualties – arising from "geological" emergencies. Among such emergencies we can name earthquakes, mudflows and earth shell dynamics. Of considerable interest are mathematical problems of a non-linear continuum, connected with mineral exploration activities. For example, in this respect, for the description of ray propagation, which is necessary for experimental geophysics (oil and gas exploration), the Finsler metric is broadly used [3–7].

1.1. Microstructure: Background for introducing non-Euclidean representations

A fundamental problem in mechanics of continua with microstructure lies in the calculation of fields of internal stresses and deformations, which arise from distributed defects of the structure. The theory of continuously distributed dislocations in solid mechanics was initially developed by Kondo [8] and independently by Bilby with co-authors [9] on the basis of the work of Nye [10]. Later, the approach was developed in the works of E. Kröner, A.M. Kosevich, and L.I. Sedov. In all these geometrical theories, the deformation tensor played the role of the fundamental metric space tensor.

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doi:10.1016/j.nonrwa.2008.09.012

Table 1
Interconnection defects field — space model.

Curvature tensors	Space model	Real object realisation
$R = S = T = 0$	Flat space	Ideal crystal
$R = T = 0,\ S \neq 0$	State of absolute parallelism	Deformed body (dislocations in continuum)
$T = S = 0,\ R \neq 0$	Riemann space	System with disclinations. Cosserat media
$R \neq 0,\ S \neq 0,\ T = 0$	Riemann–Cartan space	Theory of dislocations and disclinations
$R \neq 0,\ S \neq 0,\ T \neq 0$	Affine — metric (Finsler) space	Real inhomogeneous body. Point, linear and surface defects

In a paper [11], W. Noll exploited the remark that material properties, more specifically material uniformity, may endow simple bodies with the structure of general manifolds. Thus, in contrast with K. Kondo, B. Bulby and E. Kröner, the geometric properties are interpreted by Noll as a consequence of constitutive prescriptions.

Besides, already Kondo [12] had established that the Cartan torsion (the first Cartan curvature tensor) describes, when simulating the behaviour of solids, movement of dislocations in the crystal. This leads to identifying the curvature with the Burgers vector of the dislocation. Plastic deformation (start of plastic flow) may be considered as destabilization of a 3D body in the Euclidean space of a greater number of measures. Thus, for the presence of all types of defects, the crystal requires the existence of three non-zero Cartan tensors [13,14]. See Table 1.

From the mathematical viewpoint, any curved space should have some definite connectivity. It opens a natural way for the introduction of parallelism. Since connectivity is non-symmetrical, the resulting space is non-Riemann in the general case [15]. It may be interpreted as a lattice curvature in the presence of dislocation, while the evolving curvature is treated as the Nye curvature [10]. Ultimately, we can consider the deformation process in the following way. At the initial moment, prior to deformation, the body contains no defects and no deformations of the ideal lattice and is bound (introduced) into 3D Euclidean continuum. In this state, the 3D Euclidean geometry with Lagrange coordinates is sufficient to describe the continuum configuration. The state of the continuum after deformation is described not only by a displacement (extraction) of the continuum after deformation from the Euclidean space but also by translation of its structure (actual configuration) into the non-Euclidean one.

1.2. Mathematical background of geo-massif description

Let the metric function $F(x^i, \dot{x}^i)$ be set in the domain R of arbitrary n-dimensional space X_n. Transformation of coordinates is set by n equations:

$$x^{j'} = x^{j'}\left(x^1, \dots, x^n\right), \quad \left(j', i' = 1, \dots, n\right). \tag{1}$$

We suppose that functions x^i are at least of class C^2, i.e., continuous in all their variables and possess a continuous derivative up to the second order inclusively. We believe also that the Jacobian of the transformation (1) does not turn identically into zero, that is $\det\left(\partial x^{i'}/\partial x^i\right) \neq 0$. Then, in space X_n, the distance may be introduced between points:

$$ds = F\left(x^i, \dot{x}^i\right). \tag{2}$$

The following obvious conditions are imposed on function $F\left(x^i, \dot{x}^i\right)$ [16]:

A. Function $F\left(x^i, \dot{x}^i\right)$ is positive, if not all $\dot{x}^i$ are equal to zero simultaneously, that is $F\left(x^i, \dot{x}^i\right) > 0$ at $\sum_i \left(\dot{x}^i\right)^2 \neq 0$.

B. The quadratic form

$$F^2_{\dot{x}^i\dot{x}^j}\left(x^i, \dot{x}^i\right)\xi^i\xi^j \equiv \frac{\partial^2 F^2\left(x^i, \dot{x}^i\right)}{\partial\dot{x}^i\partial\dot{x}^j}\xi^i\xi^j > 0,$$

at all ξ^i, is not equal to zero simultaneously.

In this case, space X_n is a positively defined Finsler space.

Arising from the usual definition of distance for the Riemannian and Euclidean spaces, $ds^2 = g_{ij}x^ix^j$, condition B defines the metric tensor of tangent space T_n:

$$g_{ij} = g_{ij}\left(x, \dot{x}\right) = \frac{1}{2}\frac{\partial^2 F^2\left(x^i, \dot{x}^i\right)}{\partial\dot{x}^i\partial\dot{x}^j}, \tag{3}$$

which is, as the metric tensor should be, the tensor of second rank.

The detailed explanation of applicability of the Finsler space as the simplest generalized non-Riemannian space to geomechanics can be found in [3,5,6].

Let us consider in the following sections the process of rupture propagation in geological structures and effects related to the non-homogeneous medium structure.