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Almost periodic sequence solutions of a discrete Lotka–Volterra competitive system with feedback control ☆

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ABSTRACT

In this paper, we consider a discrete Lotka–Volterra competitive system with feedback control. Assuming that the coefficients in the system are almost periodic sequences, we obtain the existence and uniqueness of the almost periodic solution which is uniformly asymptotically stable.

1. Introduction

Recently, many scholars have paid attention to the non-autonomous discrete population models, since the discrete time models governed by difference equation are more appropriate than the continuous ones when the populations have non-overlapping generations (see [9,10,12,14–17]). Moreover, since the discrete time models can also provide efficient computational models of continuous models for numerical simulations, it is reasonable to study discrete time models governed by difference equations.

Zhou and Zou [17] had studied a discrete Logistic equation

$$x(n+1) = x(n)\exp\left\{r_1(n)\left(1-\frac{x(n)}{K_1(n)}\right)\right\}. \tag{1.1}$$

When $\{r(n)\}$ and $\{K(n)\}$ are positive ω-periodic sequences, sufficient conditions are obtained for the existence of a positive and globally asymptotically stable ω-periodic solution.

In Reference [1], Yuming Chen and Zhan Zhou had studied a discrete Lotka–Volterra competitive system

$$\begin{aligned} x(n+1) &= x(n)\exp\left\{r_1(n)\left(1-\frac{x(n)}{K_1(n)}-\mu_2(n)y(n)\right)\right\},\\ y(n+1) &= y(n)\exp\left\{r_2(n)\left(1-\mu_1(n)x(n)-\frac{y(n)}{K_2(n)}\right)\right\}. \end{aligned} \tag{1.2}$$

Sufficient conditions which guarantee the persistence of the system (1.2) are obtained. Moreover, assuming that the coefficients in the system are periodic sequences, they obtained the sufficient conditions which guarantee the existence of a globally stable periodic solution of the system (see [1,2,5,6,8–10] and the references cited therein).

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Recently, Chen and Chen [2] considered the following discrete Lotka–Volterra competitive system with feedback control:

$$\begin{aligned} x_1(n+1) &= x_1(n)\exp\left\{r_1(n)\left(1-\frac{x_1(n)}{K_1(n)}-\mu_2(n)x_2(n)-b_1(n)u_1(n)\right)\right\},\\ x_2(n+1) &= x_2(n)\exp\left\{r_2(n)\left(1-\frac{x_2(n)}{K_2(n)}-\mu_1(n)x_1(n)-b_2(n)u_2(n)\right)\right\},\\ \Delta u_1(n) &= -a_1(n)u_1(n)+c_1(n)x_1(n),\\ \Delta u_2(n) &= -a_2(n)u_2(n)+c_2(n)x_2(n). \end{aligned} \tag{1.3}$$

They first obtained the persistence of the system. Assuming that the coefficients in the system are periodic, they obtained the existence of a periodic solution. Moreover, under some additional conditions, this periodic solution is globally stable. To the best of our knowledge, though many works have been done for the Lotka–Volterra competitive system with feedback control, most of the works dealt with the continuous time model. For more results about the existence of almost periodic solutions of a continuous time system, we can refer to [3,4,7,11,13] and the references cited therein. There are few works that consider the existence of almost periodic solutions for discrete time Lotka–Volterra model with feedback control.

In this paper, based on the persistence result, under the assumptions of almost periodicity of the coefficients of system (1.3), we discuss the existence and uniqueness of almost periodic solutions for the system (1.3). Throughout this paper, we assume that

(H) $\{r_i(n)\}$, $\{K_i(n)\}$, $\{\mu_i(n)\}$, $\{a_i(n)\}$, $\{b_i(n)\}$ and $\{c_i(n)\}$ for $i = 1, 2$ are bounded non-negative almost periodic sequences such that

$$\begin{aligned} &0 < r_i^l \le r_i(n) \le r_i^u, \qquad 0 < K_i^l \le K_i(n) \le K_i^u, \qquad 0 < \mu_i^l \le \mu_i(n) \le \mu_i^u,\\ &0 < a_i^l \le a_i(n) \le a_i^u, \qquad 0 < b_i^l \le b_i(n) \le b_i^u, \qquad 0 < c_i^l \le c_i(n) \le c_i^u. \end{aligned} \tag{1.4}$$

Here, for any bounded sequence $\{a(n)\}$, $a^u = \sup_{n\in N}\{a(n)\}$ and $a^l = \inf_{n\in N}\{a(n)\}$.

By the biological meaning, we focus our discussion on the positive solutions of the system (1.3). So it is assumed that the initial conditions of (1.3) are of the form

$$x_i(0) > 0, \qquad u_i(0) > 0, \quad i = 1, 2. \tag{1.5}$$

One can easily show that the solutions of (1.3) with the initial condition (1.5) are defined and remain positive for all $n \in N^+ = \{0, 1, 2, \ldots\}$.

Definition 1.1 (*[10]*). A sequence $x : Z \to R^k$ is called an almost periodic sequence if the ε-translation set of x:

$$E\{\varepsilon, x\} := \{\tau \in Z : |x(n+\tau) - x(n)| < \varepsilon\} \quad \text{for all } n \in Z$$

is a relatively dense set in Z for all $\varepsilon > 0$, that is, for any given $\varepsilon > 0$, there exists an integer $l > 0$ such that each discrete interval of length l contains a integer $\tau = \tau(\varepsilon) \in E\{\varepsilon, x\}$ such that

$$|x(n+\tau) - x(n)| < \varepsilon \quad \text{for all } n \in Z,$$

τ is called the ε-translation number of $x(n)$.

Definition 1.2 (*[10]*). Let $f : Z \times D \to R^k$, where D is an open set in R^k, $f(n, x)$ is said to be almost periodic in n uniformly for $x \in D$, or uniformly almost periodic for short, if for any $\varepsilon > 0$ and any compact set S in D, there exists a positive integer $l(\varepsilon, S)$ such that any interval of length $l(\varepsilon, S)$ contains a integer τ for which

$$|f(n+\tau, x) - f(n, x)| < \varepsilon$$

for all $n \in Z$ and $x \in S$. τ is called the ε-translation number of $f(n, x)$.

Lemma 1.1 (*[10]*). *$\{x(n)\}$ is an almost periodic sequence if and only if for any sequence $\{h'_k\} \subset Z$ there exists a subsequence $\{h_k\} \subset \{h'_k\}$ such that $x(n + h_k)$ converges uniformly on $n \in Z$ as $k \to \infty$. Furthermore, the limit sequence is also an almost periodic sequence.*

2. Main results

For our purpose, we first introduce the following results which are given in [2,10].

Lemma 2.1 (*[2]*). *Assume that (1.4) and (1.5) hold, furthermore, $1 - \mu_1^u x_1^* - b_2^u u_2^* > 0$, $1 - \mu_2^u x_2^* - b_1^u u_1^* > 0$, then*

$$\begin{aligned} x_{i*} &\le \liminf_{n\to+\infty} x_i(n) \le \limsup_{n\to+\infty} x_i(n) \le x_i^*,\\ u_{i*} &\le \liminf_{n\to+\infty} u_i(n) \le \limsup_{n\to+\infty} u_i(n) \le u_i^*, \end{aligned} \tag{2.1}$$