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An analysis on existence and global exponential stability of periodic solutions for BAM neural networks with time-varying delays☆

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Abstract

In this paper, the problem on periodic solutions of the bidirectional associative memory neural networks with both periodic coefficients and periodic time-varying delays is discussed. By using analytic methods, inequality technique and M-matrix theory, several sufficient conditions ensuring the existence, uniqueness, and global exponential stability of periodic solution are derived. Moreover, the exponential convergence rate index is estimated, which depends on the system parameters. Some existing results are improved and extended. The obtained results are less restrictive than previously known criteria, and the hypothesis for the boundedness and monotonicity on the activation functions and the differentiability on the time-varying delays are removed. An example is given to show the effectiveness of the obtained results.

Keywords: BAM neural networks; Periodic solution; Exponential convergence; Time-varying coefficient; Time-varying delays

1. Introduction

A class of two-layer interassociative network called bidirectional associative memory (BAM) neural network is an important model with the ability of information memory and information association, which is crucial for application in pattern recognition, solving optimization problems and automatic control engineering [18,24,19]. In such applications, the dynamical characteristics of networks play an important role.

As is well known, in both biological and man-made neural networks, time delays occur due to finite switching speed of the amplifiers and communication time. The delays are usually time-varying, and sometimes vary violently with time. They slow down the transmission rate and can influence the stability of designed neural networks by creating oscillatory or unstable phenomena [3]. So it is more in accordance with this fact to study the BAM neural networks with time-varying delays. The circuits diagram and connection pattern implementing for the delayed BAM neural networks can be found in [12]. In recent years, some useful results on the uniqueness and global stability of the equilibrium

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point for the delayed recurrent neural networks and delayed BAM neural networks have been given, for example, see [1–4,6–8,10,12,13,16,17,19–21,25–30,32,34] and references therein.

It is well known that studies on neural dynamical systems not only involve a discussion of stability properties, but also involve many dynamic behaviors such as periodic oscillatory behavior, synchronization, dissipativity, bifurcation, and chaos [29,12,15,9]. In [15], the authors studied the global point dissipativity of neural networks with mixed time-varying delays. In [9], based on the invariant principle of functional differential equations, a simple adaptive feedback scheme is proposed for the synchronization of almost all kinds of coupled identical neural networks with or without time-varying delays. In many applications, the properties of periodic oscillatory solutions are of great interest, it has been found applications in learning theory [31], which is motivated by the fact that learning usually requires repetition. Hence, it is of prime importance to study periodic oscillatory solutions of neural networks. In addition, an equilibrium point can be viewed as a special periodic solution of neural networks with arbitrary period [33]. In this sense, the analysis of periodic solutions of neural networks may be considered to be more general than that of equilibrium point. Recently, periodic solution for BAM neural networks with delays has been studied, for example, see [29,12,11,5,23,22,14] and references therein. In [29,12,11,14], some sufficient conditions ensuring the existence, uniqueness and global exponential stability of periodic solution were given for BAM neural networks with constant delays. In [5,23,22], under the hypothesis for the boundedness or monotonicity on the activation functions and the differentiability on the time-varying delays, the authors gave several sufficient conditions ensuring the existence and global exponential stability of periodic solution for BAM neural networks with time-varying delays. However, in some applications, one requires to use unbounded activation functions. For example, when neural networks are designed for solving optimization problems in the presence of constraints (linear, quadratic, or more general programming problems), unbounded activations modelled by diode-like exponential-type functions are needed to impose constraints satisfaction. The extension of the quoted results to the unbounded case is not straightforward. When considering the widely employed piecewise-linear neural networks, infinite intervals with zero slope are present in activations, it is of great interest to drop the assumptions of monotonicity and continuous first derivative for the activation. Therefore, it seems that for some purposes, nonmonotonic (and not necessarily smooth) functions might be better candidates for neuron activation in designing and implementing an artificialneural network.

Motivated by the above discussions, the objective of this paper is to give some novel sufficient conditions ensuring the existence, uniqueness, and global exponential stability of periodic solution for BAM neural networks with both periodic coefficients and periodic time-varying delays, and estimate the exponential convergence rate. The method in this paper make use of neither Lyapunov functional which is used in [29,12,11,5,14] nor the continuation theorem of coincidence degree theory which is used in [23,22], and the hypothesis for boundedness and monotonicity on the activation functions and differentiability on time-varying delays are removed.

2. Model description and preliminaries

In this paper, we consider the following model:

$$\begin{cases} \dfrac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -a_i(t)u_i(t) + \sum\limits_{j=1}^{m} h_{ij}(t)f_j(\lambda_j v_j(t-\tau_{ij}(t))) + I_i(t), & i=1,2,\ldots,n, \\ \dfrac{\mathrm{d}v_j(t)}{\mathrm{d}t} = -b_j(t)v_j(t) + \sum\limits_{i=1}^{n} w_{ji}(t)g_i(\mu_i u_i(t-\sigma_{ji}(t))) + J_j(t), & j=1,2,\ldots,m \end{cases} \tag{1}$$

for $t>0$, where $u(t)=(u_1(t),u_2(t),\ldots,u_n(t))^{\mathrm{T}}\in R^n$, $v(t)=(v_1(t),v_2(t),\ldots,v_m(t))^{\mathrm{T}}\in R^m$, $u_i(t)$ and $v_j(t)$ are the state of the *i*th neurons from the neural field F_U and the *j*th neurons from the neural field F_V at time *t*, respectively; f_j, g_i denote the activation functions of the *j*th neurons from F_V and the *i*th neurons from F_U at time *t*, respectively; $I_i(t)$ and $J_j(t)$ denote the external inputs on the *i*th neurons from F_U and the *j*th neurons from F_V, respectively; $\tau_{ij}(t)$ and $\sigma_{ji}(t)$ correspond to the transmission delays and satisfy $0\leqslant\tau_{ij}(t)\leqslant\tau_{ij}$ and $0\leqslant\sigma_{ji}(t)\leqslant\sigma_{ji}$ (τ_{ij} and σ_{ji} are constants); $a_i(t)>0$ and $b_j(t)>0$ represent the rate with which the *i*th neuron from F_U and the *j*th neurons from F_V will reset their potential to the resting state in isolation when disconnected from the networks and external inputs, respectively; $h_{ij}(t)$ and $w_{ji}(t)$ denote the connection strengths; $\lambda_j>0$ and $\mu_i>0$ are constants, correspond to the neuronal gains associated with the neuronal activations [3,16].