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Nonlinear Analysis: Real World Applications 8 (2007) 1272 – 1292
Nonlinear Analysis
Real World Applications
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Codimension two bifurcations of fixed points in a class of vibratory systems with symmetrical rigid stops

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Received 30 May 2006; accepted 25 July 2006

Abstract

A vibratory system having symmetrically placed rigid stops and subjected to periodic excitation is considered. Local codimension two bifurcations of the vibratory system with symmetrical rigid stops, associated with double Hopf bifurcation and interaction of Hopf and pitchfork bifurcation, are analyzed by using the center manifold theorem technique and normal form method of maps. Dynamic behavior of the system, near the points of codimension two bifurcations, is investigated by using qualitative analysis and numerical simulation. Hopf-flip bifurcation of fixed points in the vibratory system with a single stop are briefly analyzed by comparison with unfoldings analyses of Hopf-pitchfork bifurcation of the vibratory system with symmetrical rigid stops. Near the value of double Hopf bifurcation there exist period-one double-impact symmetrical motion and quasi-periodic impact motions. The quasi-periodic impact motions are represented by the closed circle and "tire-like" attractor in projected Poincaré sections. With change of system parameters, the quasi-periodic impact motions usually lead to chaos via "tire-like" torus doubling.

Keywords: Vibration; Clearance; Periodic motion; Codimension two bifurcation

1. Introduction

Vibrating systems with clearances, gaps or stops are frequently encountered in a wide variety of engineering applications. Examples of these types of machines and equipment include gears, piping systems, heat exchangers, fuel elements of nuclear reactors and wheel–rail interaction of high speed railway coaches, etc. In the past several years, dynamics of mechanical systems with clearances, gaps or stops have been the subject of several investigations, and the large interest in analyzing and understanding the performance of such systems is reflected by vast and ever increasing amount of research effort devoted in this area, a small sample of which is reported in Refs. [1–3,5–11,13,14,18–23]. Hopf bifurcations of vibratory system with rigid stops, associated with non-resonance, weak resonance and strong resonance cases, were studied in Refs. [15–17,24,25]. However, these studies focused mainly attention on Hopf bifurcation associated with codimension one case. The purpose of the present study is to focus attention on two-parameter bifurcations of fixed points in a class of vibratory systems with symmetrical rigid stops. The paper is the continuation of Refs. [15–17,24,25] and explains in more detail dynamical behavior of the systems near the point of double Hopf

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1468-1218/$ - see front matter
doi:10.1016/j.nonrwa.2006.07.003

bifurcation and the interaction point of Hopf and pitchfork bifurcations. The results from qualitative analysis and numerical simulation show that the vibratory systems with rigid stops, near the points of codimension two bifurcations, exhibit richer and more complicated dynamical behavior.

2. The mechanical model and Poincaré map

Consider a multi-degree-of-freedom system having symmetrically placed rigid stops and subjected to periodic excitation as shown in Fig. 1. Displacements of the masses $M_1, M_2, \ldots, M_{n-1}$ and M_n are represented by $X_1, X_2, \ldots, X_{n-1}$ and X_n, respectively. The masses are connected to linear springs with stiffnesses $K_1, K_2, \ldots, K_{n-1}$ and K_n, and linear viscous dashpots with damping constants $C_1, C_2, \ldots, C_{n-1}$ and C_n. Damping in the mechanical model is assumed to be proportional damping of the Rayleigh type. The excitations on the masses are harmonic with amplitudes $P_1, P_2, \ldots, P_{n-1}$ and P_n. The excitation frequency Ω and phase angle τ are the same for these masses. For small forcing amplitudes the system will undergo simple oscillations and behave as a linear system. As the amplitude is increased, the kth mass M_k eventually begins to hit the stops and the motion becomes nonlinear (the other masses are not allowed to impact any rigid stop). The impact is described by a coefficient of restitution R, and it is assumed that the duration of impact is negligible compared to the period of the force.

The motion processes of the system, between consecutive impacts occurring at the stop A, are considered. The time T is always set to zero directly at the starting point A (the mass M_k departing from the $X_k = B$ stop with negative velocity), and the phase angle τ is used only to make a suitable choice for the origin of time in the calculation. The state of the vibro-impact system, immediately after impact, has become new initial conditions in the subsequent process of the motion. Between the stops, the non-dimensional differential equations of motion are given by

$$M\ddot{x} + C\dot{x} + Kx = F\sin(\omega t + \tau), \tag{1}$$

where M, C and K are non-dimensional mass, damping and stiffness matrices, respectively, $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$, $F = (f_{10}, f_{20}, \ldots, f_{n0})^{\mathrm{T}}$.

When the impacts occur, for $|x_k| = \delta$, the velocities of the impacting mass M_k are changed according to the impact law

$$\dot{x}_{kA+} = -R\dot{x}_{kA-}, \quad (x_k = \delta), \quad \dot{x}_{k\bar{A}+} = -R\dot{x}_{k\bar{A}-}, \quad (x_k = -\delta). \tag{2}$$

In Eqs. (1) and (2), a dot $(\cdot)$ denotes differentiation with respect to the non-dimensional time t. $\dot{x}_{kA-}$ and $\dot{x}_{kA+}$ ($\dot{x}_{k\bar{A}-}$ and $\dot{x}_{k\bar{A}+}$) represent the impacting mass velocities of approach and departure at the instant of impacting with the stop

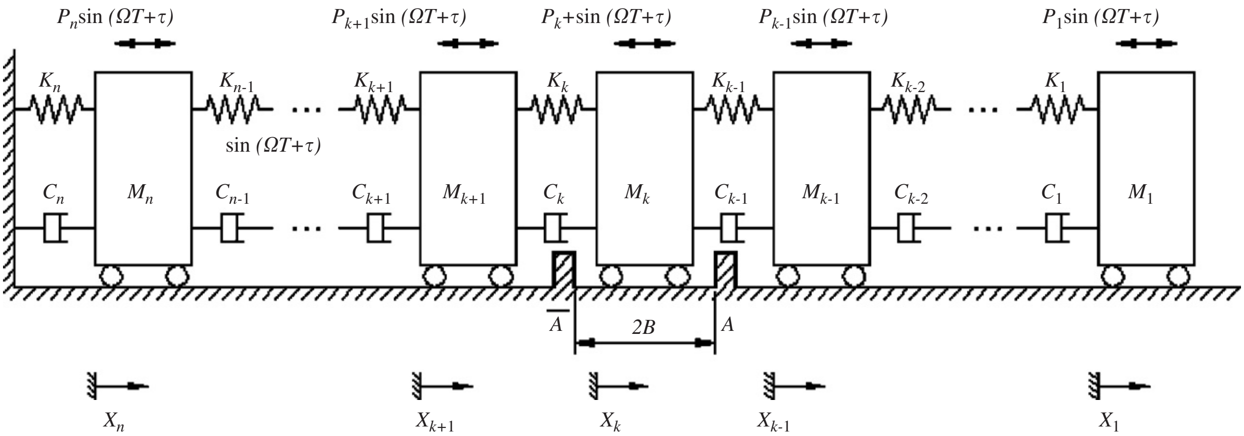

Fig. 1. Schematic of the vibratory system with symmetrical rigid stops.