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Learning dynamics in second order networks

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Abstract

A model of a second order neural network incorporating a weight adjusting learning component and time delays in processing and transmission is formulated. Delay-independent sufficient conditions are derived for the existence of an asymptotically and exponentially stable equilibrium state. The learning dynamics is modelled with an unsupervised Hebbian-type learning algorithm together with a forgetting term. If there is nothing for the network to learn or if there are no second order synaptic interactions, then our analysis will correspond to one of the standard model of neural networks.

MSC: 92B20; 34K20

Keywords: Second order networks; Hebbian learning; Asymptotic stability; Exponential stability; Lyapunov–Kravsovskii functional

1. Introduction

The subject of artificial neural networks has been vigorously studied by mathematicians, physicists, computer scientists, neurobiologists, engineers and psychologists. Hopfield-type neural networks have been intensively considered by numerous authors due to their potential applications in many areas such as pattern recognition, signal processing, function approximation, optimization, etc. It has been found that these networks have certain limitations such as limited storage capacity and inability to solve certain types of optimization problems with general constraints (see for details [1,24,4,14,32]). The class of Hopfield networks are characterized by first order interactions involving two neurons at synaptic junctions. One of the ways of enhancing the architecture of the Hopfield-type networks is to consider second or higher order interactions at the synaptic ports. From a neurophysiological viewpoint, we note that microscopic studies of neural tissues have revealed that the concept of neuron interacting through the mechanism of synaptic contact with another by means of a synaptic weight is excessively simplified and it is not uncommon to find that the axons of two or more neurons jointly make contact with a dendrite so as to form a synaptic junction of higher order than what is conceived in the formulation of the usual Hopfield model of synaptic junctions (see for instance [28]). In artificial networks, it is shown that such high order synaptic connections can improve the storage capacity [14], convergence rate and solve more general optimization problems [8,18,19]. More recently, there has been several publications concerned with the dynamics of higher order neural networks [35,36,5,10,6,34,27,21,13,23].

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An additional component of the architecture of higher order neural networks is to incorporate a learning dynamics into it and consider the combined temporal evolution of the synaptic dynamics and neuronal activations. This author is not aware of any previous work which considers the combined dynamics of synaptic modifications and neuronal activations in higher order neural networks. It is the purpose of this article to formulate a model of neural network with second order interactions involving three neurons and which includes an unsupervised Hebbian-type learning mechanism. The learning component of our model is based on the models proposed by Amari [2,3] (see also [25,26]) which is related to proposal of Hebb [12] (see also [9,17,20]).

2. Model formulation

We consider the following class of second order Hopfield-type neural networks modified by the inclusion of a learning component and processing and transmission delays

$$\left.\begin{aligned}\frac{\mathrm{d}u_i(t)}{\mathrm{d}t} &= -a_iu_i(t)+\sum_{j=1}^{n} b_{ij}g_j(u_j(t-\tau_j))\\ &\quad+\sum_{i=1}^{n}\sum_{j=1}^{n} T_{ijk}g_j(u_j(t-\tau_j))g_k(u_k(t-\tau_k))\\ &\quad+B_i\sum_{j=1}^{n} m_{ij}(t)p_j+I_i,\\ \frac{\mathrm{d}m_{ij}(t)}{\mathrm{d}t} &= -\alpha_i m_{ij}(t)+\beta_i g_i(u_i(t))p_j,\end{aligned}\right\},\quad i=1,2,\ldots,n,\quad t>0 \tag{2.1}$$

in which $u_i(t)$ denotes the membrane potential or neuronal state of the ith neuron at time t, a_i denotes a positive constant corresponding to the negative resetting feedback rate of the neuron i; $m_{ij}(t), i, j=1,2,\ldots,n$ denote the synaptic vector of the neuron i undergoing the temporal process of learning the signal vector $p=(p_1,p_2,\ldots,p_n)^{\mathrm{T}}$; also m_{ij} models the efficiency with which the ith neuron's jth synapse can transform presynaptic potential to post-synaptic potential; B_i denotes the uptake of the input signal by the ith neuron; b_{ij}, T_{ijk} denote, respectively, the first and second order synaptic weights; α_i and β_i denote disposable scaling constants with $\alpha_i>0$ and the vector $I=(I_1,I_2,\ldots,I_n)^{\mathrm{T}}$ denotes the external input signal vector stimulating the network. The neuronal activation functions $g_j(.), j=1,2,\ldots,n$ are assumed to satisfy

$$|g_j(x_j)|\leqslant M_j \quad \text{and} \quad |g_j(x_j)-g_j(y_j)|\leqslant L_j|x_j-y_j|,\quad j=1,2,\ldots,n,\quad x,y\in\mathbb{R}^n,$$

for some positive constants $M_j,\ L_j,\ j=1,2,\ldots,n$.

The second of the equations in (2.1) corresponds to an unsupervised Hebbian-type learning algorithm with a negative feedback term or forgetting term introduced so as to control the synaptic vector from becoming excessively large; such forgetting terms were not present in the original version of the Hebbian learning algorithms (see [12,17,11]). The second equation of the model system (2.1) is conjectural since it is not validated by neurophysiological evidence or experiments and such formulations have been suggested but is based on the work of Amari [2,3] which is considered recently by Meyer-Baese [25,26] and Lu and He [22]. There is a wide variety of learning algorithms used in the neural network literature. The learning dynamics incorporated in (2.1) is based on a Hebbian postulate along with a forgetting factor. We refer to Amari [3] for more details of several learning algorithms.

For convenience in the analysis of (2.1), we introduce another auxiliary state vector variable $v_i(t), i=1,2,\ldots,n$ defined by

$$v_i(t)=\sum_{j=1}^{n} m_{ij}(t)p_j,\quad i=1,2,\ldots,n,\quad t>0$$

so that the second of the equations in (2.1) becomes

$$\frac{\mathrm{d}v_i(t)}{\mathrm{d}t}=-\alpha_i v_i(t)+\beta_i g_i(u_i(t))\sum_{j=1}^{n} p_j^2.$$