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MHD flow of a micropolar fluid near a stagnation-point towards a non-linear stretching surface

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Abstract

The two-dimensional magnetohydrodynamic (MHD) stagnation-point flow of an incompressible micropolar fluid over a non-linear stretching surface is studied. The resulting non-linear system of equations is solved analytically using homotopy analysis method (HAM). The convergence of the obtained series solutions is explicitly discussed and given in the form of recurrence formulas. The influence of various pertinent parameters on the velocity, microrotation and skin-friction are shown in the tables and graphs. Comparison is also made with the corresponding numerical results of viscous ($K = 0$) [R. Cortell, Viscous flow and heat transfer over a nonlinearly stretching sheet, Appl. Math. Comput. 184 (2007) 864–873] and hydrodynamic micropolar fluid ($M = 0$) [R. Nazar, N. Amin, D. Filip, I. Pop, Stagnation point flow of a micropolar fluid towards a stretching sheet, Internat. J. Non-Linear Mech. 39 (2004) 1227–1235] for linear and non-linear stretching sheet. An excellent agreement is found.

Keywords: Micropolar fluid; MHD flow; Stagnation-point flow; Non-linear stretching; Series solution; HAM

1. Introduction

It is well known that the Navier–Stokes equations do not adequately describe the flow properties of polymeric fluids, muds, ketchup, fluids containing certain additives and some naturally occurring fluids such as animal blood. Because of the great diversity in the physical structure of non-Newtonian fluids there is not a single constitutive equation which can describe all the properties of non-Newtonian fluids. Therefore, many constitutive equations have been suggested and most of them are empirical and semi-empirical. The literature on the flows of fluids involving such constitutive equations is quite extensive. However, some very recent investigations in this direction are made in the studies [3–17].

Amongst the several models of non-Newtonian fluids, the micropolar fluids have attracted much attention from researchers. This is due to the fact that the equation governing the flow of a micropolar fluid involves a microrotation vector and a gyration parameter in addition to the classical velocity vector field. This fluid model may be applied to explain the flow of colloidal solutions, liquid crystals, suspension solutions, animal blood etc. The boundary layer concept in such a fluid past a linear stretching surface has been recently investigated by Abo-Eldahab and El Aziz [18].

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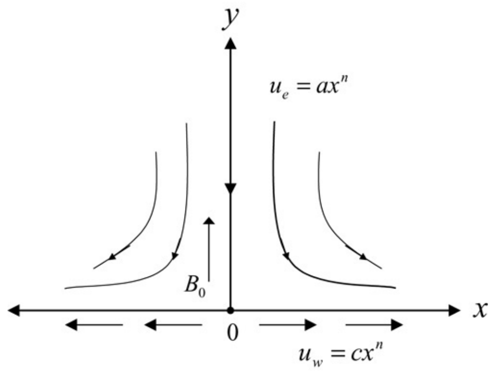

Fig. 1. Physical model and coordinate system.

Cortell [1] analyzed the hydrodynamic flow and heat transfer over a non-linear stretching sheet. Lok et al. [19] studied the boundary layer flow of a micropolar fluid starting impulsively from rest near the forward stagnation point of a plane surface. Nazar et al. [2] analyzed the stagnation-point flows of a micropolar fluids over a linear stretching sheet when the sheet is stretched in its own plane with a velocity proportional to the distance from the stagnation-point.

All the above mentioned studies of micropolar fluids have been discussed by using numerical schemes. This paper is devoted to the analytic study of the boundary layer flow of a micropolar fluid toward a stretching sheet with a stagnation-point on the plate, and tending to potential flow a infinity. The purpose of the current study is three fold (i) to consider a flow of a micropolar fluid over a non-linear stretching sheet (ii) to include the MHD effects and (iii) to provide an analytic solution. Series solutions have been obtained using a powerful technique namely the homotopy analysis method (HAM) proposed by Liao [20]. This method has been already successfully applied recently to many interesting problems [21–40]. The HAM is rather general and contains homotopy perturbation method (HPM) [41,42]. δ-expansion method and Adomian decomposition method [43]. To the best of our knowledge, the results reported here are new.

2. Formulation of the problem

Consider the steady two-dimensional, incompressible flow of a micropolar fluid near a stagnation-point in the region $y > 0$. The plane surface is located at $y = 0$ with a fixed end at $x = 0$. We take the non-linear stretching sheet in the XOZ plane (see Fig. 1). Two equal and opposite forces are applied along the x-axis. The surface is stretched in the x-direction such that the x-component of the velocity varies non-linearly along it, i.e. $u_w(x) = cx^n$, where c (>0) is constant of proportionality and n is a power index. A magnetic field of uniform strength $\mathbf{B}_0$ is applied perpendicular to the surface. The magnetic Reynolds number is taken to be very small enough so that the induced magnetic field can be neglected. It is also assumed that the ambient fluid is moved with a velocity $u_e(x) = ax^n$, where a (>0) is a constant. In the absence of body forces and body couple, the equations governing the flow of an incompressible micropolar fluid are described by:

$$\nabla \cdot \mathbf{V} = 0, \tag{1}$$

$$\rho \frac{\mathrm{D}\mathbf{V}}{\mathrm{D}t} = -\nabla p + (\mu + k)\, \nabla^2 \mathbf{V} + k \nabla \times \mathbf{N} + \mathbf{J} \times \mathbf{B}, \tag{2}$$

$$\rho j \frac{\mathrm{D}\mathbf{N}}{\mathrm{D}t} = \gamma \nabla\, (\nabla \cdot \mathbf{N}) - \gamma \nabla \times (\nabla \times \mathbf{N}) + k \nabla \times \mathbf{V} - 2k\mathbf{N}, \tag{3}$$

where $\mathrm{D}/\mathrm{D}t$ is the material derivative, $\mathbf{V}$ and $\mathbf{N}$ represent the velocity and microrotation vectors, μ is the dynamic viscosity, ρ and j denote the density and the gyration parameters of the fluid, γ and k are the spin gradient viscosity and vortex viscosity, respectively.

The equations governing the boundary layer flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{4}$$