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A regularization smoothing Newton method for solving nonlinear complementarity problem[☆]

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Abstract

In this paper, we propose a regularization smoothing Newton method for solving nonlinear complementarity problem with P_0-function (denoted by P_0-NCP(F)) based on Fischer–Burmeister function with perturbed parameter ε, which is also called smoothing parameter. The algorithm considered here has global convergence. Under suitable conditions, the method has a superlinear and quadratic convergence rate without requiring strict complementarity conditions. Moreover, at each step, we only solve a linear system of equations.

Keywords: Nonlinear complementarity problem; Regularization method; Smoothing Newton method; Perturb; Convergence

1. Introduction

The nonlinear complementarity problem (denoted by NCP(F)) [1–3] is to find a vector $x \in R^n$ such that

$$x \geq 0, \qquad F(x) \geq 0, \qquad x^{\mathrm{T}} F(x) = 0 \tag{1.1}$$

where $F : R^n \to R^n$ is a given P_0-function. Throughout this paper, we assume that F is continuously differentiable.

This problem has attracted much attention due to its various applications. We refer the reader to [1–5] for details. The methods considered here are intended to handle singular nonlinear complementarity problems, in which the derivative of the mapping F may be seriously ill-conditioned. The singularity problem will prevent most of the currently available algorithms from converging to a solution of NCP(F). In the literature there are two classes of methods that can be used to deal with the singularity nonlinear complementarity problems: regularization methods [6–8] and proximal point methods [9,10]. See the recent report of Ecktein and Ferris [11] and references therein for details. In this paper, we discuss the class of regularization methods. This class of methods try to circumvent

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the singularity problem by considering a sequence of perturbed problems which possibly have better conditions. For nonlinear complementarity problems, the simplest regularization technique is to use the so-called *Tikhonov-regularization*, which involves solving a sequence of complementarity problems NCP(F_ε):

$$x \geq 0, \quad F_\varepsilon(x) \geq 0 \quad \text{and} \quad x^{\mathrm{T}} F_\varepsilon(x) = 0 \tag{1.2}$$

where $F_\varepsilon(x) = F(x) + \varepsilon x$ and ε is a parameter converging to zero. In the paper [8], the author introduced a regularization Newton method for NCP(F_ε). In this paper we consider a regularization smoothing Newton method for solving a sequence of complementarity problems NCP(F_ε), where $F_\varepsilon(x) = F(x) + \frac{1}{2}\varepsilon \mathrm{e}^{\sin x}$ and ε is a parameter converging to zero. It is clear that F_ε is P-function if F is P_0-function. Our algorithm is based on Fischer–Burmeister smoothing function with perturbed parameter ε. For this method we prove – assuimg that F is P_0-function – that every accumulation point of the sequence of the iterations is a solution of NCP(F_ε) and that the iteration sequence is bounded if the solution set of NCP(F_ε) is nonempty and bounded. If the iteration sequence has an accumulation point and satisfies a nonsingularity conditions, then the convergence rate is global and locally superlinear(quadratic) without assuming the strict complementarity conditions. Furthermore, if the Jacobian of F is Lipschitz continuous, then the regularization smoothing Newton algorithm has global linear and local quadratic convergence. These feature can be seen clearly in the following discussion.

The organization of this paper is as follows. In the next section we study some preliminary results. In Section 3 we state the algorithm and prove several propositions related to the algorithm. In Section 4 we analyze the convergence of the algorithm. Final conclusions are given in Section 5.

The following notions will be used throughout this paper. All vector are column vectors, the subscript T denotes transpose, R^n (respectively, R) denotes the space of n-dimensional real column vectors (respectively, real numbers), R^n_+ and R^n_{++} denote the nonnegative and positive orthants of R^n, R_+(respectively, R_{++}) denotes the nonnegative(respectively, positive) orthant in R. We denote by S the solution set of (1.1). For any $\alpha, \beta \in R_{++}, \alpha = O(\beta)$ means α/β is uniformly bounded as $\beta \to 0$ $\alpha = o(\beta)$ means α/β tends to zero as $\beta \to 0$.

2. Some preliminaries

We need the following definitions concerning matrices and functions.

Definition 2.1. (1) A matrix $M \in R^n$ is said to be a P_0-matrix if all its principal minors are nonnegative. The class of such matrices is denoted P_0.

(2) A function $F : R^n \to R^n$ is said to be a P_0-function if for all $x, y \in R^n$ with $x \neq y$, there exists an index $i_0 \in N$ such that

$$x_{i_0} \neq y_{i_0}, \quad (x_{i_0} - y_{i_0})[F_{i_0}(x) - F_{i_0}(y)] \geq 0.$$

The Fischer–Burmeister function $\phi : R^2 \to R$ introduced by Fischer in [12] is defined by

$$\phi_{FB}(a, b) = a + b - \sqrt{a^2 + b^2}.$$

The function $\phi(\cdot)$ has the following important property:

$$\phi_{FB}(a, b) = 0 \Longleftrightarrow a, b \geq 0, \quad ab = 0.$$

Let $F_{\varepsilon,i}$ be the ith component of $F_{\varepsilon,i} \in N = \{1, 2, \ldots, n\}$, we have

$$\phi_{FB}(x_i, F_{\varepsilon,i}(x)) = (x_i + F_{\varepsilon,i}(x)) - \sqrt{x_i^2 + F^2_{\varepsilon,i}(x)}.$$

By $F_\varepsilon(x) = F(x) + \frac{1}{2}\varepsilon \mathrm{e}^{\sin x}$, we also have

$$\phi(\varepsilon, x, F(x)) = x + F(x) + \frac{1}{2}\varepsilon \mathrm{e}^{\sin x} - \sqrt{x^2 + \left[F(x) + \frac{1}{2}\varepsilon \mathrm{e}^{\sin x}\right]^2}. \tag{2.1}$$

It is easy to show that $\phi(a, b)$ defined by (2.1) is a smoothing Fischer–Burmeister function and $\phi(0, a, b) = \phi_{FB}(a, b)$.