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Stability analysis for periodic solution of neural networks with discontinuous neuron activations

Huaiqin Wu*

Department of Applied Mathematics, Yanshan University, Qinhuangdao 066001, China

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Abstract

In this paper, we present a general class of neural networks with discontinuous neuron activations and varying coefficients, where the neuron activation function is a discontinuous monotone increasing and bounded function. By using the fixed point theorem in differential inclusion theory and constructing suitable Lyapunov functions, a condition is derived which ensures the existence and global exponential stability of a unique periodic solution for the neural network. Furthermore, under certain conditions global convergence in finite time of the state is investigated. The obtained results show that Forti's conjecture for neural networks without delays is true. Finally, two numerical examples are given to demonstrate the effectiveness of the results obtained in this paper.

Keywords: Neural networks; Global exponential stability; Neuron activation functions; Convergence in finite time; Differential inclusions

1. Introduction

In the application of neural networks either as associative memories or as optimization solvers, the stability of networks is a prerequisite. Particularly, when neural networks are employed as associative memories, the equilibrium points represent the stored patterns, and, the stability of each equilibrium point means that each stored pattern can be retrieved even in the presence of noise. When employed as an optimization solver, the equilibrium points of neural networks correspond to possible optimal solutions, and the stability of networks then ensures the convergence to optimal solutions. Also, stability of neural networks is fundamental for network designs. Due to these, stability analysis of neural networks has received extensive attention from a lot of scholars so far [1–13]. It is well known that studies on neural networks not only involve discussions of stability property of the equilibrium point, but also involve investigations of other dynamics behaviors such as periodic oscillation, bifurcation and chaos. In many applications, knowing the property of periodic oscillatory solutions is very interesting and valuable. For example, the human brain is often in periodic oscillatory or chaos state, hence it is of prime importance to study periodic oscillatory and chaos phenomenon of neural networks for understanding the function of the human brain. In the existing literature, almost all results on the stability of periodic solutions of neural networks with or without time delays are conducted under some special assumptions on neuron activation functions. These assumptions frequently

* Tel.: +86 03358509655.
E-mail address: huaiqinwu@ysu.edu.cn.

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include those such as Lipschitz conditions, bounded and/or monotonic increasing property (see, for instance, [1–8] and the references therein). Recently, in Refs. [9–13], the authors discussed global stability for the neural networks with discontinuous neuron activations and constant coefficients. Particularly, in [13], Forti conjectures that all solutions of neural networks with discontinuous neuron activations converge to an asymptotically stable limit cycle (periodic solution) whenever the neuron inputs are periodic functions. As far as we know, there are few papers which deal with this conjecture. The purpose of this paper is, by using the fixed point theorem of differential inclusion theory and some new analysis techniques, and by constructing suitable Lyapunov functions, to study the existence, uniqueness and global exponential stability of periodic solution of neural networks with discontinuous neuron activations and varying coefficients. The conclusions obtained in this paper can be thought of as a generalization of the previous results established for neural networks possessing smooth neuron activations and periodic inputs [2–7], and neural networks with discontinuous neuron activations but without delays [11]. We have proved the Forti's conjecture in [13] for neural networks without delays is true.

For later discussion, we introduce the following notations.

Let $x = (x_1, \dots, x_n)'$, $y = (y_1, \dots, y_n)'$, $x, y \in R^n$, where the prime means the transpose. By $x > 0$ (respectively, $x \geq 0$) we mean that $x_i > 0$ (respectively, $x_i \geq 0$) for all $i = 1, \dots, n$. $\|x\| = (\sum_{i=1}^n x_i^2)^{\frac{1}{2}}$ denotes the Euclidean norm of x. $\langle x, y\rangle = \sum_{i=1}^n x_i y_i \langle\cdot,\cdot\rangle$ denotes the inner product. By the Cauchy inequality, it easily follows

$$|\langle x, y\rangle| \leq \|x\|\|y\|.$$

Given a set $Q \subset R^n$, by $K[Q]$ we denote the closure of the convex hull of Q. If $\hat{x} \in R^n$ and $r > 0$, $B(\hat{x}, r) = \{x \in R^n : \|x - \hat{x}\| < r\}$ denotes the ball with radius r and center $\hat{x}$. $\mu(Q)$ denotes the Lebesgue measure in R^n of Q. Let X be a Banach spaces, $\|x\|_X$ denotes the norm of x, $\forall x \in X$. By $L^1([0, \omega], R^n)$, $\omega \leq +\infty$, we denote the Banach spaces of the Lebesgue integrable functions $x(\cdot)$: $[0, \omega] \to R^n$ equipped with the norm $\int_0^\omega \|x(t)\|\mathrm{d}t$. Let $V : R^n \to R$ be a locally Lipschitz continuous function, The Clarke's generalized gradient [16] of V at x is defined by

$$\partial V(x) = K[\lim \nabla V(x_i) : x_i \to x,\ x_i \in R^n \setminus \Omega_V \cup \mathcal{N}],$$

where $\Omega_V \subset R^n$ is the set of Lebesgue measure zero where ∇V does not exist, and $\mathcal{N} \subset R^n$ is an arbitrary set with measure zero. For example, if $V : R \to R$ is given by $V(x) = |x|$, then we have

$$\partial V(x) = K[\mathrm{sign}(x)] = \begin{cases} 1, & x > 0, \\ [-1, 1], & x = 0, \\ -1, & x < 0. \end{cases}$$

The rest of this paper is organized as follows. In Section 2, a new neural network model considered in this paper is developed, and some preliminaries also are given. In Section 3, the proof on the existence of periodic solution for the neural network is presented. Section 4 discusses global exponential stability and convergence in finite time for the neural networks. A sufficient condition ensuring the global exponential stability for the neural network is given. Illustrative examples are provided to show the effectiveness of our results in Section 5. Some conclusions and hints are drawn in Section 6.

2. Preliminaries

The model we consider in the present paper is the neural networks modelled by the differential equation

$$\dot{x}(t) = -D(t)x(t) + B(t)g(x(t)) + I(t), \tag{1}$$

where $x(t) = (x_1(t), \dots, x_n(t))'$ is the vector of neuron states at time t; $D(t) = \mathrm{diag}(d_1(t), \dots, d_n(t))$ is $n \times n$ continuous ω-periodic diagonal matrices, $d_i(t) > 0, i = 1, \dots, n$ are the neural self-inhibitions at time t; $B(t) = (b_{ij}(t))_{n\times n}$ are an $n \times n$ continuous ω-periodic interconnection matrix; $g(x) = (g_1(x_1), \dots, g_n(x_n))'$: $R^n \longrightarrow R^n$, $g_i, i = 1, \dots, n$ represents the neuron input–output activation and $I(t) = (I_1(t), \dots, I_n(t))'$ is the continuous ω-periodic vector function denoting neuron inputs.

For the neuron activations $g_i, i = 1, \dots, n$, we assume that

H_1: (1) $g_i, i = 1, \dots, n$ is piecewise continuous, i.e., g_i is continuous in R except at a countable set of jump discontinuous points, and in every compact set of R, has only a finite number of jump discontinuous points.

(2) $g_i, i = 1, \dots, n$ is nondecreasing and bounded.