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Dynamics of an impact-progressive system

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Abstract

An impact oscillator with a frictional slider is considered. The basic function of the investigated system is to overcome the frictional force and move downwards. Based on the analysis of the oscillatory and progressive motions of the system, we introduce an impact Poincaré map with dynamical variables defined at the impact instants. The nonlinear dynamics of the impact system with a frictional slider is analyzed by using the impact Poincaré map. The stability and bifurcations of single-impact periodic motions are analyzed, and some information about the existence of other types of periodic-impact motions is provided. Since the system equilibrium is moving downwards, one way to monitor the progression rate is to calculate its progression in a finite time. The simulation results show that in a finite time, the largest progression of the system is found to occur for period-1 multi-impact motions existing in the regions of low forcing frequencies. Secondly, the progression of the period-1 single-impact motion with peak-impact velocity is also distinct enough. However, it is important to note, that the largest progression for period-1 multi-impact motion existing at a low forcing frequency is not an optimal choice for practical engineering applications. The greater the number of the impacts in an excitation period, the more distinct the adverse effects such as high noise levels and wear and tear caused by impacts. As a result, the progression of the period-1 single-impact motion with the peak-impact velocity is still optimal for practical applications. The influence of parameter variations on the oscillatory and progressive motions of the impact-progressive system are elucidated accordingly, and feasible parameter regions are provided.

Keywords: Impact; Vibration; Periodic motion; Stability; Bifurcation

1. Introduction

An impact oscillator, often named a vibro-impact system, is the term used to represent a system which is driven in some way and which also undergoes intermittent (or a continuous sequence of) contacts with motion limiting constraints. Many mechanical systems in engineering applications fall into this field. For example, the principles of the operation of vibration hammers, impact dampers, inertial shakers, pile drivers, milling and forming machines

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etc., are based on the impact action for moving bodies. With other equipment, e.g. machines with clearances, heat exchangers, steam generator tubes, fuel rods in nuclear power plants, rolling railway wheelsets, piping systems, gear transmissions and so on, impacts also occur, but they are undesirable as they bring about failures, strains, shorter service lives, and increased noise levels. It is important to be able to accurately model the dynamics of an impacting system, so as to enlarge profitable effects such as optimum design and reliability analysis and to minimize adverse effects such as pitting, scoring, and high noise levels. Compared with a single impact, the nonlinear dynamics of vibro-impact systems is more complicated. The trajectories of such systems in phase space have discontinuities caused by the impacts. Consequently, the presence of the nonlinearity and discontinuity complicates the dynamic analysis of such systems considerably, but they can nevertheless be described theoretically and numerically by discontinuities in good agreement with reality. The broad interest in analyzing and understanding the performance of such systems is reflected by an ever increasing amount of investigations devoted to this area. Several methods of theoretical analysis have been developed and different models of impacts have been assumed in the past several years. Stabilities and bifurcations of different types of impact oscillators were reported in Refs. [1–8]. Blazejczyk–Okolewska et al. [9] provided much information of a fundamental nature that broadens the scope of knowledge on the motion of mechanical systems with impacts. A special feature of impacting systems is the instability caused by grazing bifurcations. The first important work in this area was done by Nordmark [10]. This work has been further expanded by thorough investigations of two-dimensional maps, where some universal behavior has been found [11–21]. Shaw [22] studied the sticking periodic motions and sliding bifurcations of an impact oscillator with a large dissipation. Luo and Gegg [23] developed the force criteria for stick and non-stick motions in harmonically forced, friction-induced oscillators from the local theory of non-smooth dynamical systems on connectable domains. Pavlovskaia and Wiercigroch [24–27] developed a series of mathematical models of impact systems with drift and revealed the complicated dynamical behavior of such systems. Souza and Caldas [28] applied a model based algorithm for the calculation of the spectrum of the Lyapunov exponents of the attractors of mechanical systems with impacts. Peterka [29] found chaotic motion of an intermittency type in impact oscillators appearing near segments of the saddle–node stability boundaries of subharmonic motions with two different impacts in the motion period. Wagg [30, 31] analyzed the chattering impacts and rising phenomena that occur in sticking solutions of impact oscillators. Several effective methods of controlling chaos and experimental analysis have been developed. Hu [32] presented results on how to control the chaos of dynamical systems with a discontinuous vector field through the paradigm of a harmonically forced oscillator having a set-up elastic stop. The algorithms of position control of impact oscillators and the synchronization of two impact oscillators are demonstrated by Lee and Yan [33]. Souza et al. [34] proposed a feedback control method to suppress chaotic behavior in oscillators with a limited power supply. An experimental study of a base excited symmetrically piecewise linear oscillator was performed by Wiercigroch and Sin [35]. Along with the basic research into vibro-impact dynamics, a wide range of impacting models have been applied to simulate and analyze engineering systems operating within bounded dynamic responses. For example, in the wheel–rail impacts of railway coaches [36,37], vibrating hammer [38], Jeffcott rotor with bearing clearance [39–41], excited pendula with impacts [42], impact dampers [43–46], gears [47–49], etc., impacting models have proved to be useful [50].

In this paper, we focus attention on analyzing the oscillatory and progressive motions of an impact oscillator with a frictional slider. The basic function of the investigated system is to overcome the frictional force and move downwards. The best progression in an excitation period is important for the working efficiency of such systems. Based on the analysis of their oscillatory and progressive motions, we introduce an impact Poincaré map with dynamical variables defined at the impact instants. The nonlinear dynamics of the system are analyzed by using the impact Poincaré map. The stability and bifurcations of single-impact periodic motions are analyzed, and then some information about the existence of other types of periodic-impact motions is provided. In a finite time, the largest progression is found to occur for period-1 multi-impact motions existing in regions of low forcing frequencies. Secondly, the progression of a period-1 single-impact motion with a large impact velocity is also distinct enough. However, it is important to note that the largest progression for period-1 multi-impact motion is not an optimal choice for practical engineering applications. The greater the number of the impacts in an excitation period, the more distinct their adverse effects such as high noise levels and the wear and tear caused by impacts. As a result, the progression of a period-1 single-impact motion with a large impact velocity is optimal for practical applications. The influence of parameter variations on the oscillatory and progressive motions of the system is elucidated, and feasible parameter regions are provided accordingly.