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Nonlinear Analysis: Real World Applications 10 (2009) 756–778
Nonlinear Analysis
Real World Applications
www.elsevier.com/locate/nonrwa

Dynamic analysis and suppressing chaotic impacts of a two-degree-of-freedom oscillator with a clearance

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Received 7 October 2007; accepted 1 November 2007

Abstract

A two-degree-of-freedom impact oscillator is considered. The maximum displacement of one of the masses is limited to a threshold value by the symmetrical rigid stops. Impacts between the mass and the stops are described by an instantaneous coefficient of restitution. Dynamics of the system is studied with special attention to periodic-impact motions and bifurcations. Period-one double-impact symmetrical motion and transcendental impact Poincaré map of the system is derived analytically. Stability and local bifurcations of the period-one double-impact symmetrical motions are analyzed by using the impact Poincaré map. The Lyapunov exponents in the vibratory system with impacts are calculated by using the transcendental impact map. The influence of the clearance and excitation frequency on symmetrical double-impact periodic motion and bifurcations is analyzed. A series of other periodic-impact motions are found and the corresponding bifurcations are analyzed. For smaller values of clearance, period-one double-impact symmetrical motion usually undergoes pitchfork bifurcation with decrease in the forcing frequency. For large values of the clearance, period-one double-impact symmetrical motion undergoes Neimark–Sacker bifurcation with decrease in the forcing frequency. The chattering-impact vibration and the sticking phenomena are found to occur in the region of low forcing frequency, which enlarges the adverse effects such as high noise levels, wear and tear and so on. These imply that the dynamic behavior of this system is very rich and complex, varying from different types of periodic motions to chaos, even chattering-impacting vibration and sticking. Chaotic-impact motions are suppressed to minimize the adverse effects by using external driving force, delay feedback and feedback-based method of period pulse.

Keywords: Vibration; Impact; Periodic motion; Bifurcation; Suppressing chaos

1. Introduction

Vibrating systems with clearances between the moving parts are frequently encountered in technical applications. Impacts occur when the amplitudes of vibration of some parts of the system exceed critical values. Some examples are linkage machines, heat exchangers, fuel rods in nuclear power plants, rolling railway wheelset, piping systems, gear transmissions and so on. Impacts inevitably occur for the vibrating systems with clearances or gaps, but they

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1468-1218/$ - see front matter
doi:10.1016/j.nonrwa.2007.11.002

are undesirable as they bring about failures, strain, shorter service life and increased noise levels. It is important to be able to accurately model the dynamics of an impacting system, so as to enlarge the profitable effects such as optimum design and high reliability, to minimize adverse effects such as high noise levels, pitting, wear and tear and so on. Compared with single impact, the nonlinear dynamics of vibro-impact systems is more complicated. The trajectories of such systems in phase space have discontinuities caused by the impacts. Consequently, the presence of the nonlinearity and discontinuity complicates the dynamic analysis of such systems considerably, but they can be described theoretically and numerically by discontinuities in good agreement with reality. The broad interest in analyzing and understanding the performance of such systems is reflected by a still increasing amount of investigations devoted to this area. Several methods of the theoretical analysis have been developed and different models of impacts have been assumed in the past several years. Stability and bifurcations of different types of impact oscillators were reported in Refs. [1–11]. Blazejczyk-Okolewska et al. [12] provided much information of the fundamental nature that broadens the scope of knowledge on the motion of mechanical systems with impacts. A special feature of impacting systems is the instability caused by grazing bifurcation. The first important work in this area was done by Nordmark [13]. This work has been further expanded by thorough investigations of two-dimensional maps, where some universal behavior has been found [14–23]. Based on the local singularity theory, Luo and Gegg [24] provided a comprehensive investigation of grazing motions in the dry-friction oscillator for a better understanding of the grazing mechanism of such a discontinuous system. Pavlovskaia and Wiercigroch [25–28] developed a series of mathematical models of impact systems with drift and revealed very complex dynamical behavior of such systems. Souza and Caldas [29] applied a model-based algorithm for the calculation of the spectrum of the Lyapunov exponents of attractors of mechanical systems with impacts. Peterka [30] found chaotic motion of an intermittency type of the impact oscillator appearing near segments of saddle-node stability boundaries of subharmonic motions with two different impacts in motion period. Luo [31] presented an idealized, piecewise linear system to model the vibration of gear transmission systems, and predicted analytically the periodic motions in a generalized, piecewise linear oscillator with perfectly plastic impacts. Wagg [32,33] analyzed the chattering impact and rising phenomena which occur in the stick solutions of impact oscillators. Luo and Gegg [34] developed the force criteria for stick and non-stick motions in harmonically forced, friction-induced oscillators from the local theory of non-smooth dynamical systems on connectable domains. Several effective methods of controlling chaos and experimental analysis have been developed in Refs. [35–42]. Hu [36] presented how to control the chaos of dynamical systems with discontinuous vector field through the paradigm of a harmonically forced oscillator having a set-up elastic stop. Lee and Yan [37] studied the algorithms of position control of impact oscillator and demonstrated synchronization of two impact oscillators. Souza et al. stabilized desired unstable periodic orbits, embedded in the chaotic invariant sets of mechanical systems with impacts, by applying a small and precise perturbation on an available control parameter [38], and investigated the influence of structural nonlinearity of a simple cantilever beam impacting system on its dynamic responses close to grazing incidence by means of numerical simulation [39]. Zinjade and Mallik [40] designed and fabricated an experimental setup of a single-degree-of-freedom friction-driven oscillator with an attached impact damper, and investigated the control of friction-driven oscillations by using an impact damper. The effects of mass ratio, coefficient of restitution and clearance on the performance of the impact damper have been investigated analytically and the results are verified by numerical integration. Zhao and Wang [41] proposed several feedback control methods to suppress chaotic behavior of an impact damper. Experimental study of base excited symmetrically piecewise linear oscillator was performed by Wiercigroch and Sin [42]. Along with the basic research into vibro-impact dynamics, a wide range of impacting models have been applied to simulate and analyze engineering systems operating within bounded dynamic responses. For example, in wheel–rail impacts of railway coaches [43,44], vibrating hammer [45], pile driver [46], Jeffcott rotor with bearing clearance [47–49], excited pendula with impacts [50], impact dampers [51–55], gears [31,56,57], etc., impacting models have proved to be useful [58].

A two-degree-of-freedom impact oscillator with a clearance is considered in the paper. The maximum displacement of one of the masses is limited to a threshold value by the symmetrical rigid stops. Dynamics of the system is studied with special attention to periodic-impact motions and bifurcations. Period-one double-impact symmetrical motion and transcendental impact map of the system are derived analytically. Stability and local bifurcations of the period-one double-impact symmetrical motions are analyzed by using the impact Poincaré map. The Lyapunov exponents in the impact oscillator with a clearance are calculated by using the transcendental impact map. The influence of the clearance and excitation frequency on the dynamics of the system is analyzed. The bifurcation regularity of period-one double-impact symmetrical motion is revealed. The chattering-impact vibration and the sticking phenomena are