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Hopf bifurcation analysis in a fluid flow model of Internet congestion control algorithm

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Abstract

This paper focuses on the Hopf bifurcation analysis of some classes of nonlinear time-delay models, namely fluid flow models, for the Internet congestion control algorithm of TCP/AQM networks. Using tools from control and bifurcation theory, it is proved that there exists a critical value of communication delay for the stability of the network. When the delay passes through the critical value, the system loses its stability and a Hopf bifurcation occurs. Furthermore, the stability of the bifurcation and direction of the bifurcating periodic solutions are determined by applying the normal form theory and the center manifold theorem. Finally, some numerical examples are given to verify the theoretical analysis.

Keywords: Hopf bifurcation; Congestion control; Time-delay system; Stability; Nonlinear dynamics

1. Introduction

Internet congestion control is an algorithm to allocate available resources to competing sources efficiently so as to avoid congestion collapse. The whole Internet congestion and avoidance mechanism is a combination of the end-to-end TCP congestion control mechanism [1] at the end hosts and the queue management mechanism at the routers. The basis of TCP congestion control lies in the Additive Increase Multiplicative Decrease (AIMD) mechanism that halves the congestion window for every window containing a packet loss, and increases the congestion window by roughly one segment per Round Trip Time (RTT) otherwise [2]. The queue management mechanism is meant to control the congestion level at each router through different kinds of AQM mechanisms, e.g. Drop Tail [1], Random Early Detection (RED) [3], Random Early Marking (REM) [4], Virtual Queue (VQ) [5], and Adaptive Virtual Queue (AVQ) [6].

Understanding the dynamics and stability of the congestion control algorithm in the Internet has been the focus of intense research in the last few years. Chaotic behavior of TCP has been reported in [7]. Using some discrete-time models, researchers have shown that TCP/RED systems develop chaotic dynamics with variability in RED

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parameters [8–12]. The local and global stability of the congestion control algorithm with or without communication delays have been studied in [13–25]. Meanwhile, Hopf bifurcation analysis of Internet congestion control systems has also drawn much attention from researchers. Using a gain parameter as a bifurcation parameter, Li et al. have studied Hopf bifurcation in a one-order REM model [26]. In [27], the authors have proved that the REM algorithm exhibited Hopf bifurcation when choosing the delay as a bifurcation parameter. Later, Raina studied the local bifurcation of the fair dual with proportional and TCP fairness, and the delay dual algorithms [28]. In [29], Raina and Heckmann have analyzed a fluid model of TCP with an approximation of Drop Tail using tools from control and bifurcation theory. Guo et al. were interested in the Hopf bifurcation of a new AQM, namely exponential RED, and considered the communication delay as a bifurcation parameter [30].

In this paper, we focus on the bifurcation analysis of a fluid flow model for TCP/AQM networks. The fluid flow model that describes accurately the behavior of congested routers of TCP/AQM networks was first introduced in [31]. Such a kind of model is described by nonlinear differential equations with a time-delay, where the delay represents the corresponding RTT in the network. Since the delay varies depending on the network's congestion status, the system may exhibit complex behaviors in practice. Here we choose the communication delay as the bifurcation parameter. From theoretical analysis we can see that there exists a critical value for this delay and the whole system is stable when the delay of the system is less than the value. Moreover, we prove that a Hopf bifurcation occurs when conditions of local stability are just violated. By applying the normal form theory and the center manifold theorem, the direction of the Hopf bifurcation and the stability of bifurcating periodic solutions are also determined.

The rest of the paper is organized as follows. By analyzing the corresponding characteristic equation of the linearized equation, the existence of the Hopf bifurcation of a fluid flow model in the Internet congestion control algorithm (with a communication delay) is investigated in Section 2. In Section 3, based on the normal form theory and the center manifold theorem, we derive the formulas for determining the properties of the direction of the Hopf bifurcation and the stability of bifurcating periodic solutions. Simulations are given in Section 4 to verify the theoretical results. Finally, conclusions are drawn in Section 5.

2. Hopf bifurcation in a fluid flow model

In [31], a dynamic model of TCP/AQM networks was introduced using fluid-flow and stochastic differential equation analysis. Here we use a simplified version of that model, which ignores the time-out and slow start mechanisms of TCP. The model consists of the following coupled nonlinear differential equations with a time-varying delay:

$$\begin{cases} \dot{W}(t) = \dfrac{1}{R(t)} - \dfrac{W(t)}{2}\dfrac{W(t-R(t))}{R(t-R(t))}p(t-R(t)) \\ \dot{q}(t) = N(t)\dfrac{W(t)}{R(t)} - C \end{cases} \tag{1}$$

where $W(t)$ denotes the average of TCP windows size (packets), $q(t)$ is the average of queue length (packets), $p(\cdot)$ is the probability function of a packet mark, $N(t)$ is the number of TCP sessions, C is the queue capacity (packets/s) and $R(t)$ is the Round Trip Time (s) which consists of the propagation delay and queuing delay. When the queuing delay is much smaller than the propagation delay, we can assume that $N(t) = N$ and $R(t) = R$ are constants [15]. Also in [16], it is considered that the probability marking function $p(\cdot)$ is proportional to the queue length, i.e. $p(t) = K \cdot q(t)$. This results in the following close-loop system:

$$\begin{cases} \dot{W}(t) = \dfrac{1}{R} - \dfrac{W(t)W(t-R)}{2R}Kq(t-R) \\ \dot{q}(t) = \dfrac{N}{R}W(t) - C. \end{cases} \tag{2}$$

Additionally, in [16] it has been shown that the Eq. (2) can be approximated by:

$$\begin{cases} \dot{W}(t) = \dfrac{1}{R} - \dfrac{W(t)W(t)}{2R}Kq(t-R) \\ \dot{q}(t) = \dfrac{N}{R}W(t) - C \end{cases} \tag{3}$$