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Application of the differential transformation method to the free vibrations of strongly non-linear oscillators

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Abstract

This paper adopts the differential transformation method to obtain the free vibration behavior of an oscillator with fifth-order non-linearities. The principle of differential transformation is briefly introduced, and is then applied in the derivation of a set of difference equations for the free vibration oscillator problem. The solutions are subsequently solved by a process of inverse transformation. The time responses of the oscillator are presented under different parameter conditions, and the current results are then compared with those derived from the established Runge–Kutta method in order to verify the accuracy of the proposed method. It is shown that there is excellent agreement between the two sets of results. This finding confirms that the proposed differential transformation method is a powerful and efficient tool for solving non-linear problems.

Keywords: Differential transformation method; Non-linear oscillators

1. Introduction

The last few decades have seen the publication of several articles relating to the non-linear free vibrations of a beam. The governing equations associated with the vibration of strings, beams and many other physical systems are known to contain fifth-order non-linearities. The aim of the current study is to obtain the free vibrations of a non-linear oscillator with fifth-order non-linearities [1] by means of the differential transformation method.

The differential transformation method is based on the Taylor's series expansion, and provides an effective numerical means of solving linear and non-linear initial value problems. A review of the related literature reveals that Chiou [2] exploited the inherent ability of differential transforms to solve non-linear problems. The differential transformation method may be employed to solve both ordinary and partial differential equations. For example, Chen and Ho [3,4] solved the free vibration problems using differential transforms. In their study of 2001, Jang, Chen and Liu [5] successfully applied the two-dimensional differential transformation method to the solution of partial differential equations. Yu and Chen [6,7] applied the differential transformation method to optimize the rectangular fin with variable thermal parameters. Finally, Hassan [8] adopted the differential transformation method to solve some

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Table 1
Specific functions, $x(t)$, and their corresponding differential transforms, $X(k)$

Original function $x(t)$	Transformed function $X(k)$
$x(t) = y(t) \pm z(t)$	$X(k) = Y(k) \pm Z(k)$
$x(t) = \alpha y(t)$	$X(k) = \alpha Y(k)$
$x(t) = \frac{\mathrm{d}y(t)}{\mathrm{d}t}$	$X(k) = \frac{k+1}{H} Y(k+1)$
$x(t) = y(t)z(t)$	$X(k) = \sum_{\lambda=0}^{k} Y(\lambda)Z(k-\lambda)$
$x(t) = \frac{y(t)}{z(t)}$	$X(k) = \frac{Y(k)-\sum_{\lambda=0}^{k-1} X(\lambda)Z(k-\lambda)}{Z(0)}$

eigenvalue problems. In general, the previous studies have demonstrated that the differential transformation method is an efficient technique for solving non-linear or parameter-varying systems.

The current study uses the differential transformation method to investigate the behavior of the free vibrations of a non-linear oscillator with fifth-order non-linearities. This paper illustrates how the corresponding non-linear equations may be converted into differential transforms and then solved by a process of inverse transformation. A comparison of the present results with those yielded by the established Runge–Kutta method confirms the accuracy of the proposed method.

2. Differential transformation method

Let $x(t)$ be an analytic function in a domain D and let $t = t_i$ represent any point in D. The function $x(t)$ is then represented by a power series whose center is located at t_i. The Taylor series expansion function of $x(t)$ is expressed as

$$x(t) = \sum_{k=0}^{\infty} \frac{(t-t_i)^k}{k!} \left[\frac{\mathrm{d}^k x(t)}{\mathrm{d}t^k} \right]_{t=t_i} \quad \text{for } \forall t \subset D. \tag{1}$$

The particular case of Eq. (1) when $t_i = 0$ is referred to as the Maclaurin series of $x(t)$, and is given by

$$x(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left[\frac{\mathrm{d}^k x(t)}{\mathrm{d}t^k} \right]_{t=0} \quad \text{for } \forall t \subset D. \tag{2}$$

As shown by Zhou [9], the differential transform of function $x(t)$ is defined as

$$X(k) \equiv \frac{H^k}{k!} \left[\frac{\mathrm{d}^k x(t)}{\mathrm{d}t^k} \right]_{t=0} \quad k = 0, 1, 2, \ldots, \infty \tag{3}$$

where $X(k)$ represents the transformed function (commonly referred to as the T-function) and $x(t)$ is the original function. The differential spectrum of $X(k)$ is confined within the interval $t \in [0, H]$, where H is a constant.

The differential inverse transform of $X(k)$ is defined as follows:

$$x(t) = \sum_{k=0}^{\infty} \left(\frac{t}{H} \right)^k X(k). \tag{4}$$

From the above, it is clear that the differential transformation technique is based upon the Taylor series expansion. Note that the original functions are denoted by lower case letters, while their transformed functions (i.e. their T-functions) are indicated by the corresponding upper case letters.

The values of function $X(k)$ at specific values of the argument k are referred to as discretes, i.e. $X(0)$ is known as the zero discrete, $X(1)$ as the first discrete etc. The greater the number of discretes considered, the more precisely the unknown function can be restored. The function $x(t)$ is expressed in terms of the T-function $X(k)$, and its value is given by the sum of the T-functions using $(t/H)^k$ as its coefficient.

Table 1 presents some important properties of the Taylor differential transformation derived using the expressions presented in Eqs. (3) and (4) above.