Available online at www.sciencedirect.com
ScienceDirect
Nonlinear Analysis: Real World Applications 10 (2009) 1151–1159
Nonlinear Analysis
Real World Applications
www.elsevier.com/locate/nonrwa

Adaptive synchronization for two identical generalized Lorenz chaotic systems via a single controller

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Received 28 October 2007; accepted 17 December 2007

Abstract

This paper presents a systematic design procedure to synchronize two identical generalized Lorenz chaotic systems based on a sliding mode control. In contrast to the previous works, this approach only needs a single controller to realize synchronization, which has considerable significance in reducing the cost and complexity for controller implementation. A switching surface only including partial states is adopted to ensure the stability of the error dynamics in the sliding mode. Then an adaptive sliding mode controller (ASMC) is derived to guarantee the occurrence of the sliding motion even when the parameters of the drive and response generalized Lorenz systems are unknown. Last, an example is included to illustrate the results developed in this paper.

Keywords: Adaptive sliding mode control; Synchronization; Chaos; Generalized Lorenz chaotic system

1. Introduction

Over the last two decades, since the pioneering work of Pecora and Carroll in 1990 [1], synchronization of chaotic systems has attracted increasing attention in different fields. Generally speaking, designing a system to mimic the behaviour of another chaotic system is called synchronization. The two chaotic systems are generally called drive (master) and response (slave) systems, respectively. Chaos synchronization can be applied widely in the fields of physics and engineering systems, such as in power converters, chemical reactions, biological systems, information processing, and especially for secure communication [2–6]. To date, different techniques and methods have been proposed to achieve chaos synchronization such as impulsive control [7,8], adaptive control [9,10], sliding mode control [11–14], fuzzy control [15], optimal control [16], digital redesign control [17], backstepping control [18,19], and so on.

The Lorenz system is one of the paradigms of chaos, since it exhibits a wide variety of nonlinear dynamics phenomena such as bifurcations and chaos. Recently Lü et al. [20] have proposed the generalized Lorenz chaotic system (GLCS) to bridge the gap between the Lorenz system and the Chen system. The solution bounds of GLCS are investigated based on the time-domain approach [21]. In this paper, the synchronization of two identical GLCSs with unknown parameters is considered.

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1468-1218/$ - see front matter
doi:10.1016/j.nonrwa.2007.12.005

The purpose of this paper is the development of an adaptive sliding mode controller (ASMC) for synchronizing the state trajectories of two identical GLCSs. A switching surface, which only includes partial states but makes it easy to guarantee the stability of the error dynamics in the sliding mode, is first proposed. And then, based on this switching surface, an ASMC is derived to guarantee the occurrence of the sliding motion, even when the drive and response systems parameters are unknown. It is worthy of note that Liao et al. [22] coped with the similar problem for a special generalized Lorenz canonical system with feedback control laws. But the control design is based on the well-known system parameters, and the control aim considered in their paper cannot be achieved if only a single control input is used. However, the approach in this paper uses only a single controller to realize synchronization, no matter that the system's parameter is unknown, which has great significance for reducing the cost and complexity for controller implementation. Finally, we present numerical simulation results to illustrate the effectiveness of the proposed control scheme.

This paper is organized as follows. Section 2 describes the dynamics of the GLCS and formulates the chaos synchronization problem. In Section 3, based on the sliding mode control, a switching surface design is first presented and the stability of the error dynamics system in the sliding mode is derived. Then a controller is proposed to achieve the hitting. In Section 4, we show an illustrative example. Finally, conclusions are presented in Section 5. Throughout this paper, it is noted that $|w|$ represents the absolute value of w and $\mathrm{sign}(s)$ is the sign function of s, if $s > 0$, $\mathrm{sign}(s) = 1$; if $s = 0$, $\mathrm{sign}(s) = 0$; if $s < 0$, $\mathrm{sign}(s) = -1$.

2. System description and problem formulation

In this section, we consider the adaptive synchronization of two identical GLCSs with a sliding mode controller.

2.1. Generalized Lorenz chaotic systems

In this paper, we consider the following GLCS:

$$\begin{aligned}
\dot{x}_1(t) &= \left(10 + \frac{25}{29}k\right) \cdot [x_2(t) - x_1(t)] \\
\dot{x}_2(t) &= \left(28 - \frac{35}{29}k\right) x_1(t) + (k-1)x_2(t) - x_1(t)x_3(t) \\
\dot{x}_3(t) &= \left(-\frac{8}{3} - \frac{1}{87}k\right) x_3(t) + x_1(t)x_2(t) \\
\begin{bmatrix} x_1(0) & x_2(0) & x_3(0) \end{bmatrix}^{\mathrm{T}} &= \begin{bmatrix} x_{10} & x_{20} & x_{30} \end{bmatrix}^{\mathrm{T}}
\end{aligned} \tag{1}$$

where $x(t) = \begin{bmatrix} x_1(t) & x_2(t) & x_3(t) \end{bmatrix}^{\mathrm{T}} \in R^3$ is state vector, $\begin{bmatrix} x_{10} & x_{20} & x_{30} \end{bmatrix}^{\mathrm{T}}$ is the initial value vector, and k is the system parameter with $0 \leqslant k < 1$. Obviously, the original Lorenz system is a special case of system (1) with $k = 0$. The dynamics of this system has been extensively studied in [20] for a space range of the amplitude of the term k and displays chaotic behaviour for each $0 \leq k < 1$. Fig. 1(a)–(d) show the chaotic motion of system (1) for $k = 0.2$ with initial condition of $\begin{bmatrix} x_{10} & x_{20} & x_{30} \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}}$. In the following, we will consider the synchronization of two identical GLCSs and give an explicit and simple procedure to establish an ASMC to achieve the control goal.

2.2. Synchronization problem formulation

Consider the following two identical GLCSs, where the drive system and response system are denoted with x and y, respectively.

Drive system:

$$\begin{aligned}
\dot{x}_1(t) &= \left(10 + \frac{25}{29}k\right) \cdot [x_2(t) - x_1(t)] \\
\dot{x}_2(t) &= \left(28 - \frac{35}{29}k\right) x_1(t) + (k-1)x_2(t) - x_1(t)x_3(t) \\
\dot{x}_3(t) &= \left(-\frac{8}{3} - \frac{1}{87}k\right) x_3(t) + x_1(t)x_2(t).
\end{aligned} \tag{2}$$