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Stability analysis for a nonlinear model of a hydraulic servomechanism in a servoelastic framework

A. Halanay[a,*], C.A. Safta[b], F. Ursu[c], I. Ursu[c]

[a] Department of Mathematics I, University Politehnica of Bucharest, Splaiul Independenţei 313, RO-060042, Bucharest, Romania
[b] Department of Hydraulic and Hydraulic Machinery, University Politehnica of Bucharest, Splaiul Independenţei 313, RO-060042, Bucharest, Romania
[c] Elie Carafoli National Institute for Aerospace Research, Bd. Iuliu Maniu 220, RO-061126, Bucharest, Romania

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Abstract

The effects of mounting structure stiffness on mechano-hydraulic servomechanisms actuating aircraft primary flight controls are modeled by a six-dimensional nonlinear system of ordinary differential equations. Stability analysis of equilibria reveals the presence of a critical case that is handled through the use of the Lyapunov–Malkin theorem. Stability charts are drawn using the Routh–Hurwitz criterion for the stability of a fifth-degree polynomial. Comparison with previous results shows how the stability of equilibria can be ensured exploiting the positive influence of structural feedback.

Keywords: Hydraulic servomechanism; Servoelasticity; Lyapunov stability; Routh–Hurwitz criterion

1. Introduction

Hydraulic servomechanisms are encountered in various industries where heavy objects are manipulated and large forces or torques at high speeds are exerted. Features such as large processing force and stiffness, high payload capabilities and good positioning and power-to-weight ratio, make this type of actuation system appropriate for the positioning of aircraft control surfaces (flight controls), high-power industrial machinery, position control of military gun turrets and antennas, material handling, construction, agricultural equipment etc. However, because of the complexity of their mathematical models, both the analysis and the design of hydraulic servomechanisms are still difficult and immature, although various mathematical and engineering methodologies were brought into the proof in this field—from the classical linearization [1], to the artificial intelligence techniques [2].

The present paper aims to study the influence of the mounting structure of an airplane on hydraulic-powered control. Such an approach belongs to the aeroservoelasticity area that studies the interactions between the aeroelastic

* Corresponding author.
E-mail addresses: halanay@mathem.pub.ro (A. Halanay), carmen123safta@yahoo.com (C.A. Safta), fursu@aero.incas.ro (F. Ursu), iursu@aero.incas.ro (I. Ursu).

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mounting structures of the aircraft and the (electro- or mechano-) hydraulic servomechanisms (the control-powered elements).

Consideration of the (aero)servoelastic features in the hydraulic servomechanism synthesis, already present in the early days of the theory (see [3]), has been approached in several papers since [4–8] and emphasized by Aviation Regulations [9]. It is worth mentioning that, during the qualification tests of powered hydraulic servos for aircraft's primary flight controls actuation, a simulated mass overload on the test rig has generated, as a response to an accidentally impulse-type disturbance at the mechanical input, a strong self-excited regime of the entire test ring (see [10]). It is the main goal of this paper to give, under a realistic nonlinear model, stability conditions that prevent appearance of such phenomena during exploitation.

Historically, a three state variables mathematical model of a servomechanism was considered (from [1,3,11,12] until [13]). It resulted from the hypotheses of very small displacements of load around the midstroke position of the piston. This amounts to considering

$$p_s = p_1 + p_2 \tag{1.1}$$

where p_1, p_2 and p_s are hydraulic oil pressures (see Fig. 1).

As a simplified valve-piston equation one takes

$$S\dot{z} + k_c \dot{p}_L = c_d w\sigma \sqrt{\frac{p_s - p_L \operatorname{sgn}\sigma}{\rho}} \tag{1.2}$$

where $p_L = p_1 - p_2$ and $k_c = \frac{V}{2B}$ (the notations are explained in Section 4).

The following linear model, based on (1.2)

$$S(\dot{z} - \dot{z}_e) + k_c \dot{p}_L = k_Q \sigma - k_{Q_p} p_L, \qquad \sigma = -\lambda_2 z + \lambda_3 z_e, \qquad S p_L = -E z_e \tag{1.3}$$

is used to provide an antiflutter stability condition

$$\lambda_3 > \left(1 + \frac{k_c E}{S^2}\right)\left(\lambda_2 - \frac{k_{Q_p}}{k_Q}\frac{ES}{S^2 + k_c E}\right). \tag{1.4}$$

In the above, E is mounting structure stiffness, λ_2, λ_3 are kinematic coefficients, k_{Q_p}, k_Q are described in Section 4; see [7,14].

Even if the three state variables models were widely used in classical analysis and synthesis of hydraulic servos in the last 40 years, it was clear from the beginning that in a realistic model both pressures in the chambers of the cylinder must be present as independent state variables [15–23]. The two pairs of equations derived from the pressure-flow equation of the spool-type valve (Bernoulli equation) and the equation of flow conservation on both sides of the piston are: for $\sigma \geq 0$

$$\begin{aligned} S\dot{z} + \frac{V + Sz}{B}\dot{p}_1 &= c_d w\sigma\sqrt{\frac{2(p_s - p_1)}{\rho}} \\ -S\dot{z} + \frac{V - Sz}{B}\dot{p}_2 &= -c_d w\sigma\sqrt{\frac{2p_2}{\rho}} \end{aligned} \tag{1.5}$$

and, for $\sigma < 0$,

$$\begin{aligned} S\dot{z} + \frac{V + Sz}{B}\dot{p}_1 &= c_d w\sigma\sqrt{\frac{2p_1}{\rho}} \\ -S\dot{z} + \frac{V - Sz}{B}\dot{p}_2 &= -c_d w\sigma\sqrt{\frac{2(p_s - p_2)}{\rho}}. \end{aligned} \tag{1.5$'$}$$

As pointed out in [3], page 82, condition (1.1) is a consequence of applying to Eqs. (1.5), (1.5′) the postulate "if the (valve) orifices are both matched and symmetrical then the flows in diagonally opposite arms of the (hydraulic) bridge